A TEXT-BOOK

OF

ELECTRICAL MACHINERY.

VOLUME I.

ELECTRIC, MAGNETIC, AND ELECTRO-STATIC CIRCUITS.

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PREFACE.

The student of electrical engineering comes to the technical and professional part of his course well grounded in the principles of elementary and applied mathematics and in the relations and characteristics of physical phenomena. His next task is to learn to apply this training to the working principles of engineering, both those underlying the design and operation of electrical machinery and those upon which general engineering methods are based. With these facts in mind it has been found desirable to produce a text for the purpose of communicating to the student the working principles mentioned above and to prepare him for reading profitably the literature of his profession. The book has been designed as a distinctively engineering text, not as a work on physics or applied mathematics. At the same time it has been found desirable to restate in engineering terms the elementary laws and principles of those sciences which bear directly upon the subject in hand.

As a result of experience in teaching electrical engineering it has been found most satisfactory, both in maintaining the interest of the student and in economizing his time and energy, to found the treatment upon the laws of the alternating-current circuit, from which the treatment of continuous-current phenomena follows naturally. The application of these laws is illustrated by means of a few problems. It has not, however,

been the purpose to make this a problem book, and the teacher and student should prepare additional problems for class and home usc. In this part of the work, books of the nature of " Electrical Problems," by Hooper and Wells, will be found of service.

Volume I covers the laws of the electric, magnetic, and electrostatic circuits in such a manner that the analyses of the structural and performance characteristics of electrical machinery, treated in Volume II, may be easily followed.

The authors express obligations to all contributors to the literature of the profession. Whi e the method of no one author has been followed, the aim has been to profit by the work of all and to provide an in:roductory text wherewith to prepare the student to secure most profitably further training from professional text, laboratories, lectures, and the unlimited sources of electr:cal-engineering personnel and literature.

The material of Volume I when amplified with additional problems, preferably taken from practical cases, may be thoroughly covered in fifty recitations, the two volumes being designed to provide profitable work for approximately one hundred class exercises.

The form of the material in this volume is the result of several years of experience in its use as a text for the instruction of classes in Cornell University.

ITHACA, NEW YORK, Sept. 1, 1903.

CONTENTS.

TABLE OF IMPORTANT SYMBOLS AND ABBREVIATIONS.

B. . . . .density of magnetic flux or induction.
C. . . . .electrostatic capacity.
e.m.f. . .electromotive force in volts.
E. . . . .e.m.f. effective value.
e. . . . .e.m.f. instantaneous value.
F. . . . .mechanical force.
f. . . . . .frequency in cycles per second.
H. . . . .magnetomotive force in gilberts.
I. . . . . .current in amperes, effective value.
i. . . . . .current, instantaneous value.
j. . . . . . $\sqrt{-1}$.
L. . . . .inductance in henrys.
m.m.f. .magnetomotive force.
μ. . . . . .magnetic permeability.
P. . . . .electric power.
Φ . . . .total magnetic induction or flux.
Q. . . . .quantity of electricity.
r. . . . . .electric resistance.
$\mathcal{R}$. . . . .magnetic reluctance.
t. . . . . .time in seconds.
θ. . . . . .angle of phase difference.
W. . . . .electric energy or work.
x. . . . . .reactance.
z. . . . . .impedance.

This table contains only those symbols and abbreviations which are frequently used. Those which are used locally only are explained when used.

ELECTRICAL MACHINERY.

CHAPTER I.

ELECTRICITY AND MAGNETISM.

SYNOPSIS.

1. Electricity and electrical energy.
2. Electromotive force. Three methods for maintaining an e.m.f. Measurement of e.m.f.
3. Magnetism:
   - *a.* Magnetomotive force.
   - *b.* Magnetic flux. Water flow.
   - *c.* Tension of the magnetic field.
   - *d.* Other hydraulic analogies to magnetism.
4. Magnetic tension and flux density.

1. Electricity and Electrical Energy.—Electrical phenomena are manifestations of molecular action. There are, unfortunately, no means available for observing the exact character of the molecular mechanisms upon which these phenomena depend. Electricity must, for this reason, be studied like heat, light, chemical energy, and other forms of molecular energy, that is, by its effects. By observation of the results of the operation of molecular forces, as manifested in mass motion or in chemical action, some idea of the forces can be gained.

As implied in the preceding statement, electricity is a form of energy. This energy has the same character as has com-

plete mass motion, which is shown by the fact that it may be readily transformed into any other kind of energy. It may also be transferred from point to point by the use of suitable molecular kinematic connection, and it is this ability to transfer power without mass motion which makes it the only successful carrier of energy over long distances. The transformation of electrical energy is electrical work, and the rate of this action is electrical power, just as in the case of mass motion.

Electrical energy may be stored in the production of magnetism just as mechanical energy is stored in accelerating the velocity of a mass. It may be recovered from its stored form. Likewise electrical energy becomes potential energy when an electric charge is taken up by capacity, just as mechanical energy is stored by elasticity in compressing a spring. This phenomenon is also reversible. These illustrations point to the identity of electrical and mechanical energies, and the important points in the study of electrical engineering are: (1) *the conservation of all energy;* (2) *the energy character of electricity.*

2. Electromotive Force. — Electromotive force * is the initial cause of the electric current and of electrostatic attraction.

This e.m.f. may be maintained by one of three methods as follows:

a. Thermo-electric.

b. Chemico-electric.

c. Dynamo-electric.

a. When the junction of two metals is heated, an e.m.f. is produced, the value of which depends on the metals and the temperature to which their junction is heated. This e.m.f. may be used to cause a flow of electric current by connecting

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* Usually written e.m.f.

the unheated terminals and thus part of the applied heat energy may be utilized electrically. But practically the portion of heat so transformed is very small, and because of this lack of economy the thermal couple is very little used. Fig. 1

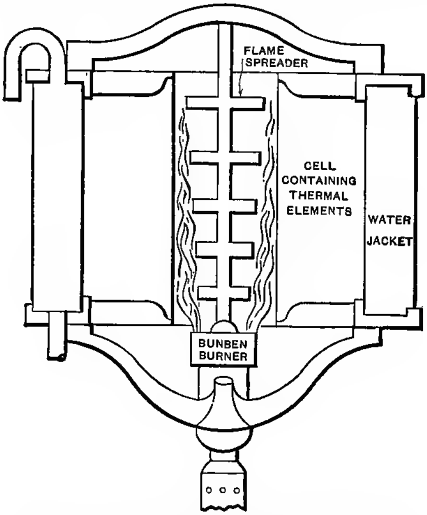


FIG. 1.—Cox Thermo-electric Generator.

represents a commercial form of the electric thermo-pile. It is known as the Cox generator.

b. If two unlike metals, not in metallic contact, are placed in a bath of some liquid which attacks one of them more than the other, an e.m.f. is set up between the metals, and by suitable connection outside the liquid an electric current may be produced. This chemical generation of current has its practical application in the primary battery which has an important place in small work. Fig. 2 shows a form of primary cell which is in common use.

c. The third method for the generation of an electromotive force is the all-important one to the engineer, and it consists in the application of the principle that a wire moved in a magnetic field in such a direction as to cut across the magnetic

flux of the field will have produced in it an electromotive force the value of which will depend on the length of wire, its

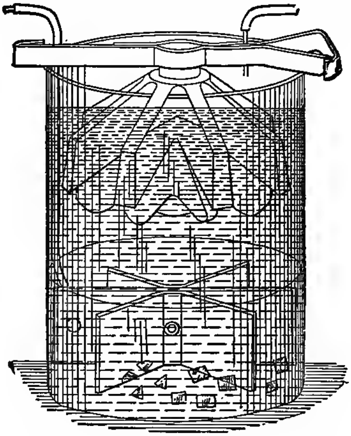


FIG. 2.—A Typical Primary Cell.

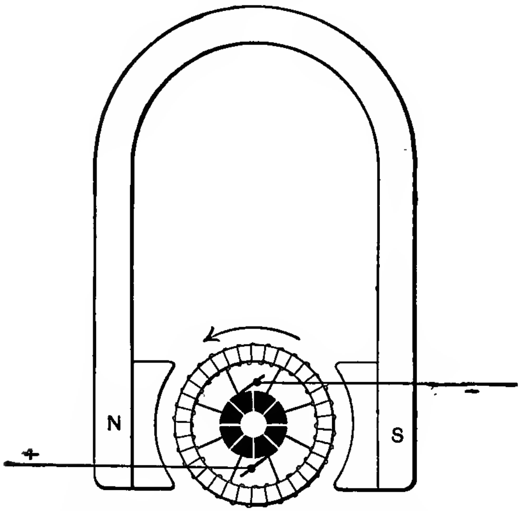


FIG. 3.—Simple Dynamo-electric Machine.

velocity, and the strength of the field cut by it. Fig. 3 illustrates experimentally this method for electro-mechanically developing an e.m.f. This principle is used in the construction

of all dynamo-electric machines which form the main means for the conversion of mechanical into electrical energy, and *vice versa*.

Measurement of e.m.f.—The presence of an e.m.f., or difference of potential, is indicated and its amount may be measured by means of an electrostatic voltmeter. This instrument, which is also known as an electrometer, is shown in a commercial form in Fig. 4. It utilizes the facts that there is repulsion or attraction between two electrically charged bodies and that two bodies may be charged by connection to the terminals of a circuit in which a difference of potential exists. In the measurement of low electrical pressures * the electrometer plates constituting the charged bodies are numerous in order that the loss of attraction due to the low pressure may be made up. In this form the instrument is known as a multicellular voltmeter.

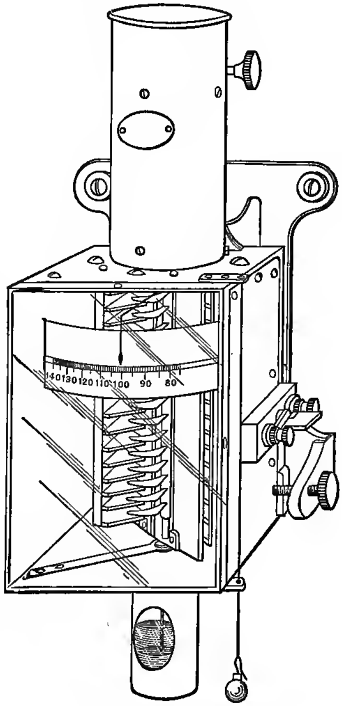


FIG. 4.—Multicellular Electrostatic Voltmeter.

3. **Magnetism.**—In Fig. 5, *NS* is a permanent bar magnet. It is made of hardened crucible steel and has been magnetized through some natural means, such as contact with another bar magnet, or with a piece of loadstone, or it has been placed in a solenoid carrying an electric current. When

* The expression "electric pressure" is often used for e.m.f.

this bar is remote from other magnetic substances and the immediate region about it is examined with a small compass, a magnetic field, such as that illustrated, will be found. This field is due to magnetism or magnetic flux, which emanates from one end of the bar and returns to the other as the lines show. Extensive experimental researches conducted by physicists have led to the following conclusions in this connection:

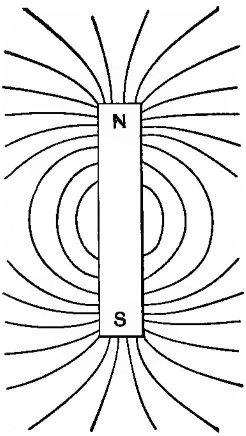


FIG. 5.—Magnetic Field surrounding a Bar Magnet.

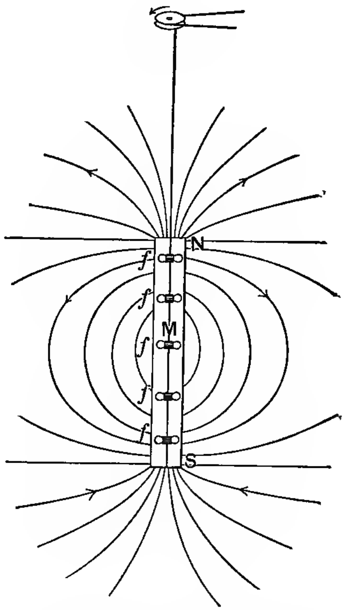


FIG. 6.—Hydraulic Model of the Magnetic Field of a Bar Magnet.

a. The magnetism about the bar magnet is due to a magnetomotive force (m.m.f.) that resides in the molecules of the bar magnet. Such m.m.f. is proportional to the length of the bar, and, therefore, to the number of molecules which constitute a single filament of the bar.

b. A m.m.f. sets up a difference of magnetic pressure

between the two ends of the bar, which causes magnetism or magnetic flux to be established from one end of the bar to the other. In Fig. 6 is an illustration of an hydraulic model of a permanent bar magnet. The model of the magnet proper is *M*, located in a vessel of comparatively large size containing water. The part *M* represents the bar magnet. It is a metal tube perforated on all sides with numerous small holes. A rotating shaft carrying screw propellers, *ffff*, furnishes the model with a water-motive force directed from *S* to *N* within the tube. This force corresponds to the magneto-motive force that resides in the molecules of the bar magnet. When the propellers, *ffff*, are set in motion, the water will enter the tube at *S* and go out at *N*. The flux of this water represents the magnetic flux. The lines drawn in the figure represent the direction of the water flux at all points in the immediate region of the tube, while the space between these lines is a measure of the cross-section over which a definite rate of water flux occurs.

In the same manner, then, as in this model the lines in Fig. 5 show at once the direction of magnetic flux and its amount at any point in the region of the magnet.

c. At all points within the field of flux there exists a mechanical force related to the magnet. The nature of this force is as follows: The magnetic flux possesses a mechanical tension along its own direction and a mechanical pressure everywhere at right angles to the direction of the magnetic flux. The entire field of flux is rigidly attached to the magnetic body from which it emanates or by which it is established. It has been found by experimental means, to be described later, that this tension is proportional to the square of the rate of magnetic flux at any point. The lateral pressure of magnetic flux is so intimately associated with the tension that exists along its own direction that it is hardly necessary to

distinguish between the two. It is necessary to keep in mind their difference, however, much as it is necessary in mechanics to keep in mind the difference between action and reaction. All engineering problems in magnetism are solved in terms of the tension of the magnetic flux.

d. Physical experience has shown further that magnetic flux is established in a closed circuit just as the water currents in the hydraulic model flow in closed circuits; i.e., whatever amount of water current enters the S end of the tube also passes out at the N end. An entirely analogous property has been found to exist for the magnetism about a magnetic body. Whatever amount of magnetic flux is emitted from one side or end, called pole, precisely that same amount re-enters at the opposite pole. This amount is everywhere in existence *en route* from one pole to the other. Of the nature of magnetic flux within the magnetic body little is known. Researches, however, have long since proven that within the magnetic body there exists a state of things corresponding to a continuity of the magnetic flux whereby a complete circuit of such flux is always established, just as is the case with the electric current in the closed electric circuit. Because our knowledge of the state of things within the magnetic body ceases at this point, that which completes the circuit of magnetic flux within and through the magnetic body is called **induction.**

4. **Magnetic Tension and Flux Density.**—In Fig. 7, AAA is a bar of soft wrought iron formed as shown, and the bar magnet NS is mounted in front of AAA and in contact with it much as shown in the figure. Under these circumstances almost all evidences of magnetic flux will be found to have disappeared in the region that surrounds NS. This is due to the fact that the magnetic flux has

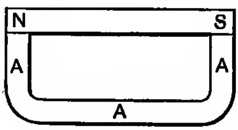


FIG. 7.—Magnet and Keeper.

disappeared *as induction* in the soft wrought iron. It has been found that but a very small amount of magnetic pressure is consumed in maintaining the induction in the iron bar, while the balance of the magnetic pressure generated by the m.m.f. that resides in the steel bar is used up point by point, simultaneously with its origin, in maintaining the induction through the steel bar *NS*. Thus it is that practically no difference of magnetic pressure results along *NS*, which accounts for the disappearance of the magnetic flux. It is true, too, that the total amount of induction which exists at any cross-section of the circuit *NSAAA* is practically the same as that which exists at any other cross-section.

In Fig. 8 the magnet *NS* is mounted on a knife-edge at *S*, and the end *N* is suspended from the stirrup of a scale-beam

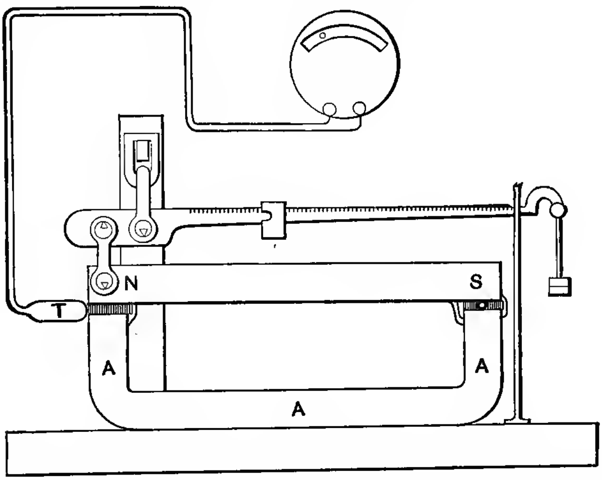


FIG. 8.—Apparatus for Studying the Tension due to Magnetic Induction.

with another knife-edge as shown. The mounting of *NS* is so adjusted that a small air-gap, *ag*, is formed separating *N* from *A*. The following facts may be observed experimentally with this apparatus:

1. *The existence of induction* in *NS* and *AAA*, and a corresponding amount of magnetic flux across the air-gaps at *N* and *S*, will at once be shown by the tension registered on the scale-beam.

2. *The tension is proportional to the square of the rate at which magnetic flux is distributed over a given cross-section.*

An indication of the amount of flux from the pole *N* into the face of the soft iron armature or keeper, *AAA*, is given by the throw of the galvanometer needle when the turn of wire, *T*, is drawn away from the position shown in the figure so as to cut the magnetic flux. The tension of the lines of flux is weighed on the scale-beam and found to be *P*. Another bar is now used in the place of the one upon which the

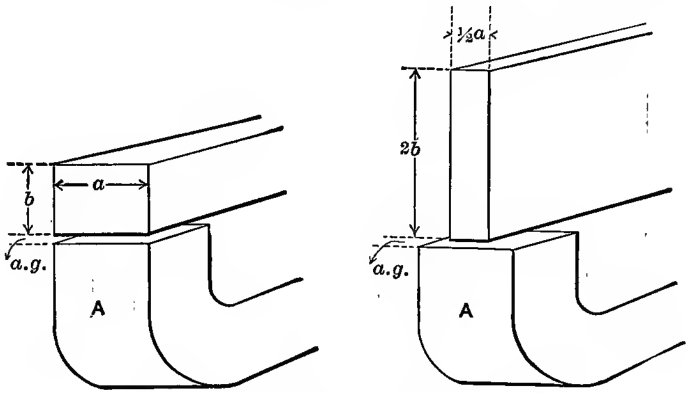


FIG. 9. FIG. 10.

Illustrating the Effect of Change of Air-gap Area.

measurement has just been made. The second bar has the same cross-section and length as the first, but instead of being square in cross-section its horizontal thickness is one-half and its vertical thickness is twice the thickness of the bar that has just been removed. That is, it is set on edge as shown in Fig. 10, and the area of the air-gaps is, therefore, one-half that found in the preceding case, which is illustrated in Fig. 9.

The m.m.f. of the bar has been adjusted so as to set up the same flux as before. This may be shown by the kick given by the galvanometer when the turn of wire, T, cuts across the flux at the air-gap. Upon weighing the tension of the flux we find it to be $2F$. Note the significance of these experiments.

First experiment:

Total flux, Φ.
Cross-section at air-gap, A.
Flux density, B', $= \dfrac{\Phi}{A}$.
Observed magnetic pull, F.

Second experiment:

Total flux, Φ.
Cross-section at air-gap, $\frac{1}{2}A$.
Flux density, B'', $= \dfrac{\Phi}{\frac{1}{2}A} = 2B'$.
Observed magnetic pull, $2F$.

Had the area in this experiment been A instead of $\frac{1}{2}A$, and had the magnetic density been maintained at $B'' = 2B'$, the magnetic pull would evidently have been $4F$. Thus we find when A remains constant,

Flux density $= B'$; *the magnetic pull* $= F$.
" " $= 2B'$; " " " $= 4F$.

If the flux density had been increased in a third experiment to $3B'$ we should have found the magnetic pull to be $9F$. Thus we learn experimentally that the contractile tension of the magnetic flux is proportional to the square of the flux density. The numerical value of this magnetic pull, F, is KB^2, where K is a constant arbitrarily chosen. Its numerical value in the centimeter-gram-second system is $\dfrac{1}{8\pi}$.

CHAPTER II.

FUNDAMENTAL AND DERIVED UNITS.

SYNOPSIS.

5. Fundamental units.
   - *a.* The unit of magnetic flux.
   - *b.* The unit of current.
   - *c.* The unit of electromotive force.
6. The electric circuit.
   - *a.* Through dynamo and simple conductor.
   - *b.* Through dynamo, conductor, and condenser.
   - *c.* Through dynamo, conductor, and electrolytic cells.
7. Derived units.
   - *a.* The unit of resistance.
   - *b.* The unit of inductance.
   - *c.* The unit quantity of electricity.
   - *d.* The unit of capacity.
   - *e.* The unit of power.
   - *f.* The unit of energy.
8. Power consumption in electric circuits.
   1. Power consumed by resistance.
   2. Power consumed by counter e.m.f.
9. Problems in the use of electrical units.

5. **Fundamental Units.**—The dynamic character of electromagnetic action is so much in accord with common mechanical experience that the electromagnetic actions have been chosen to form the basis for the definition of a system of absolute electrical units,—often called the centimeter-gram-second (*c.g.s.*) system. This system is universally adopted in electrophysics and in electrical engineering. The *c.g.s.* units are usually of inconvenient magnitude and a system of practical units is necessary. The practical units are arbitrary multiples of the *c.g.s.* units.

a. The unit of magnetic flux.

Practical unit, the Maxwell, equal to the c.g.s. unit, is the flux which will produce a tension in its own direction of $1 \div 8\pi$ *dynes when distributed uniformly over one sq. cm. of cross-section.*

The name of this unit of magnetic flux is the **maxwell**. The *density* of magnetic flux is the number of *maxwells per unit cross-section.*

b. The unit of current.

Practical unit, the Ampere: one-tenth of the c.g.s. unit.

From experimental research it has been found that a straight conductor carrying an electric current in a uniform field of magnetic flux will be acted upon by a mechanical force tending to move it at right angles to the direction of the flux. It has been found that this force is proportional to the length of the conductor in the flux, the current strength, and the flux density. These facts form the basis for defining the unit of current strength as follows:

One unit of current in a wire located in and at right angles to a uniform field of unit flux density will cause a mechanical force of one dyne to be applied to each centimeter length of the conductor, at right angles both to the flux and to the conductor.

This is the *c.g.s.* unit. One-tenth of it has been adopted as a convenient unit for practical purposes. The name of this practical unit of current is the **ampere**.

c. The unit of electromotive force.

Practical unit, the Volt: one hundred million c.g.s. units.

Experimental research has determined that an e.m.f. is generated in a conductor moved across a field of magnetic flux that is proportional to the velocity, flux density, and length of the conductor moving through the flux. On the basis of these facts the value of the unit of e.m.f. is determined by definition as follows:

One unit of electromotive force is generated in each centimeter length of a conductor moving at a velocity of one centimeter per second through a uniform field of unit flux density and at right angles both to the flux and to the length of the conductor. This is a convenient definition, but the size of the resulting unit is inconveniently small for practical purposes. The practical unit that has been adopted, because of its convenient size, is one hundred million (10^8) times the size of the *c.g.s.* unit as determined by definition. The name of this practical unit is the **volt**.

There are other electric and magnetic units. They are derived from the above fundamental units, among which must be included the unit of time, or the **second**. These derived units will be discussed in connection with the control of the electric current and the magnetic flux.

6. The Electric Circuit.—In no case is electric action possible unless there is a complete circuit over which a transfer of an electric charge may occur. This circuit must be complete through the source of e.m.f. as well as by an external route.

a. In Fig. 11 the dynamo forms a part of the conducting electric circuit in which the actuating e.m.f. is generated. The current is established by this e.m.f. through the complete circuit of the dynamo and conductor as indicated in the figure.

FIG. 11.—Simple Conductor Circuit.

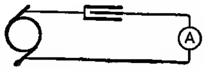


FIG. 12.—Circuit containing Condenser.

b. In Fig. 12 the circuit may be closed through the dynamo, the conductor, and a condenser. Under these circumstances a rush of current, as indicated by the instrument at *A*, will occur at the instant the circuit is closed. As soon as the strain in the dielectric of the condenser will no longer

increase under the pressure of the actuating e.m.f., all current through the condenser will cease. Now if the conductor terminals are reversed by some such means as illustrated in Fig. 13, a momentary current will be shown on the instrument *A* while the process of relieving the dielectric strain in one direction and applying it in the other is going on. Continued reversal of the condenser terminals by revolving the commutator in Fig. 13 will cause a succession of current impulses to be established in the circuit.

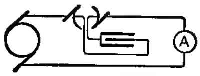


FIG. 13.—Circuit containing Condenser with Commutator.

FIG. 14.—Circuit containing Electrolytic Cells.

c. Again, as in Fig. 14, the circuit may be established through the dynamo, an electric conductor, and one or more electrolytic cells. In general the cells will transmit the current with more or less irregularity through processes of internal molecular changes or transfers. Electrolytic cells do not merely close an electric circuit as does a conductor or a condenser; they are generally sources of e.m.f. and thus modify the current by subtracting from or adding to the source of e.m.f.

7. Derived Units.—*Current Control.*—The current that will be set up in any electric circuit depends upon the value of the source e.m.f., and upon the values of the resistance, capacity, and inductance of the circuit. Capacity and inductance present phenomena like resilience and mass motion in mechanics.

a. THE UNIT OF RESISTANCE.

Practical unit, the Ohm: 10^9 *c.g.s. units.*

It is found experimentally that when a current is established in a conductor at any constant temperature, electric pressure is consumed in direct proportion to the value of the current.

This property of a conductor is called electric resistance. Based on the above fact, the definition of the unit of resistance is as follows:

One unit of resistance will consume one unit of pressure per unit of current.

The practical unit of resistance is called the **ohm.** It consumes *one volt per ampere.* On account of the ampere the ohm is ten times and on account of the volt it is a hundred million (10^8) times, or a total of one thousand million (10^9) times, the magnitude of the *c.g.s.* unit.

b. THE UNIT OF INDUCTANCE.

Practical unit, the Henry: 10^9 c.g.s. units.

Every conductor carrying current sets up magnetic flux about itself. This has been established by experiment. In the same way it is learned that such flux cannot be brought into existence by the current without cutting across the conductor about which it is established. In doing so an e.m.f. is generated in the conductor that is equal to the rate at which the magnetic flux cuts across it. As long as the current is changing, an e.m.f. is self-generated in the conductor. *The direction of this self-induced e.m.f. is always such as to oppose the corresponding current change.* The process of setting up magnetic flux about a conductor by the current it carries is called *self-induction*, and the ability to self-generate an e.m.f. is given the name *inductance.* The above facts constitute the basis for defining the unit of inductance as follows:

A circuit possesses one unit of inductance when a unit rate of change of current in the circuit generates one unit of e.m.f.

Since this is a derived unit, the magnitude of the corresponding practical unit is at once determined by reference to the ampere and the volt. The second always remains as the practical unit of time. The ampere is one-tenth of, and the volt one hundred million (10^8) times the corresponding absolute

unit. This would make the practical unit of inductance one thousand million (10^9) times the absolute unit determined by definition. The name of this practical unit of inductance is the **henry.**

c. THE UNIT QUANTITY OF ELECTRICITY.

Practical unit, the Coulomb, one-tenth c.g.s. unit.

When a dielectric is subjected to electric pressure a definite strain is produced. In the production of this strain a quantity of electricity must be applied by transfer through the circuit. A unit for quantity of electricity is, therefore, necessary.

One unit quantity of electricity is equal to the quantity transferred by one unit of current in one unit of time.

This unit is derived from the fundamental units. Its corresponding practical value is the ampere-second. On account of the ampere it is one-tenth of the value of the absolute unit. The practical unit quantity of electricity is called the **coulomb.**

d. THE UNIT OF CAPACITY.

Practical units, the Farad, 10^{-9} c.g.s. unit, and the Microfarad, 10^{-15} c.g.s. unit.

If an electric circuit be closed through a condenser, some or all of the impressed pressure of the electric circuit will be taken up by the capacity of the dielectric. Experiment reveals in this connection the following fact: The dielectric constituting the capacity will take up a quantity of electricity, or electric charge, in proportion to the electric pressure applied between its faces. As this applied pressure is changed, the electric charge accepted by the dielectric is correspondingly changed. The rate of transfer of electric charge is the value of the electric current by which the change is accomplished. The unit of capacity is, therefore, defined in terms of the units of current, pressure, and time, thus:

A dielectric in an electric circuit has a capacity of unity

when the transfer through it of unit current requires a unit rate of change of the applied pressure.

The magnitude of the corresponding practical unit of capacity, therefore, becomes one tenth on account of the ampere, and one hundred-millionth (10^{-8}) on account of the volt, making the practical unit one thousand-millionth (10^{-9}) of the absolute unit. The name of this practical unit of capacity is the **farad.** The farad is inconveniently large for most practical purposes, so that condensers are ordinarily rated in a unit that is one millionth of the farad. This substitute for the farad is called the **microfarad.** The microfarad is, therefore, one million-thousand-millionth (10^{-15}) of the absolute unit.

e. THE UNIT OF POWER.

Practical unit, the Watt: 10^7 c.g.s. units.

It has been found by experiment that the power in any part of an electric circuit is proportional to the product of the e.m.f. at its terminals and the current present.

The unit of power is applied when a unit of current is established by a unit of pressure.

The corresponding practical unit of power is the volt-ampere. The name of this unit is the **watt.** Being derived from the ampere and the volt it is, therefore, one-tenth on account of the ampere and one hundred million (10^8) times on account of the volt, or ten million (10^7) times the absolute unit. *It has a mechanical equivalent of .0013405 horse-power; i.e., there are 746 watts in one horse-power.*

f. THE UNIT OF ENERGY.

Practical unit, the Joule: 10^7 c.g.s. units.

Power is the rate of transformation of energy from one form to another. The amount of energy thus transformed is the product of the power and the time.

The unit of energy is transformed by unit power in unit time.

The corresponding practical unit is the watt-second (10^7 *c.g.s.*). The name of this unit is the **joule.**

8. Power Consumption in Electric Circuits. — By the consumption of power in an electric circuit is meant the transformation of electrical energy into some other form. This transformation may occur in two ways:

1. *Power Consumed by Resistance.*—The power that is consumed by the resistance of a conductor is IE, where I is the current in the conductor and E the pressure used in establishing the current. By definition the value of the resistance in ohms means the number of the volts used per ampere in setting up the current in the conductor. It follows, then, that

$$E = Ir.$$

The power consumed by the resistance of the wire is, therefore,

$$W = IE = I^2 r.$$

This power is lost from the wire as heat. The electric power changed to heat in a conductor due to its resistance is proportional to the square of the current.

The relation $E = Ir$, as above determined, may be transposed so as to stand

$$I = \frac{E}{r}, \quad \dots \dots \dots \quad (1)$$

when it becomes **Ohm's law**, which states:

In a closed electric circuit the current equals the ratio of the electromotive force to the resistance of such circuit.

2. *Power Consumed by Counter-Electromotive Force.*

When electrical energy is stored in or about an electric circuit or is transformed into mechanical or chemical energy,

an e.m.f. is produced in the circuit with a direction opposed to the current. Such an e.m.f. is called a **counter-electromotive force.**

The rate at which the electrical energy is stored or transformed is equal to the product of the counter-electromotive force and the current.

The energy of a circuit may be transformed into heat by dielectric and magnetic molecular action in and about the circuit. Such processes are substantially the same as the dissipation of heat in resistance.

9. **Problems in the Use of Electrical Units.**—Prob. 1. If the space between two poles, A and B, Fig. 15, is a magnetic field and the area of each pole is 10 sq. in., find the pull between the poles in pounds when the density of magnetism in the field is 10,000 maxwells per sq. cm. One pound is 445,000 dynes. 576.9 lbs. *Ans.*

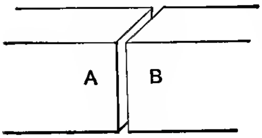


FIG. 15.

Prob. 2. In the magnetic circuit shown in Fig. 16 there is a density of magnetism of 10,000 maxwells per sq. cm ($B =$ 10.000). What should be the area of each pole in sq. ins.: (*a*) To produce a pull of 100 lbs.? (*b*) To produce a pull of 50 lbs.?

(*a*) .8667 sq. in. *Ans.*
(*b*) .4333 " " "

Prob. 3. If the armature of the magnet shown in Fig. 16 exposes 10 sq. in. surface to each pole and weighs 10 lbs., what density of induction per sq. cm. is needed to support a weight of (*a*) 50 lbs.? (*b*) 100 lbs.? (*c*) 200 lbs.?

(*a*) 2280 maxwells per sq. cm. *Ans.*
(*b*) 3088 " " " " "
(*c*) 4266 " " " " "

Prob. 4. The wire shown in Fig. 17 carries 10 amperes.

The total magnetism, uniformly distributed over the poles, is 1,000,000 maxwells. The poles are 3 in. square. What force, measured in pounds, urges the wire across the field?

.295 lb. *Ans.*

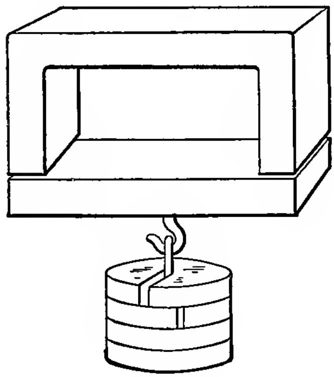


FIG. 16.

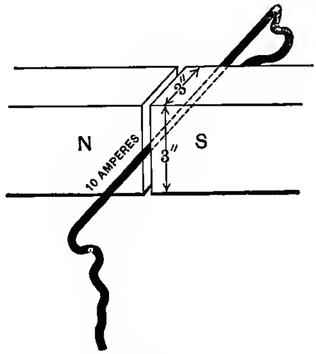


FIG. 17.

Prob. 5. A wire in a uniform magnetic field makes an angle of 45° with a plane normal to the field. When 10 amperes flow through the wire and the induction density is 50,000 maxwells per sq. in., what force, measured in pounds per inch of length of the conductor, urges it across the field?

.0313 lb. *Ans.*

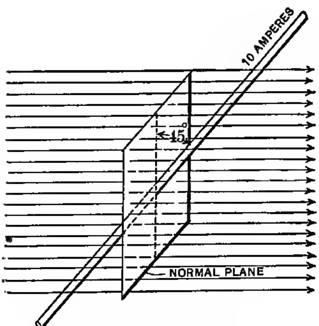


FIG. 18.

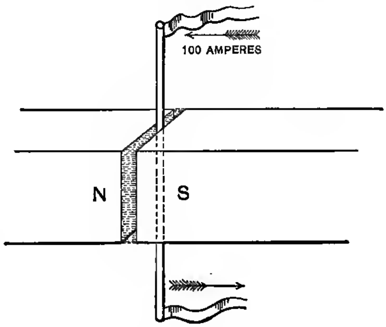


FIG. 19.

Prob. 6. How much work in ft.-lbs. will be done in passing a wire carrying 100 amperes through the magnetic field shown

in Fig. 19, when 10,000,000 maxwells of flux exist between the poles: (*a*) When the poles are 10 in. square? (*b*) When the poles are 5 in. square?

(*a*) 7.373 ft.-lbs. *Ans.*
(*b*) 7.373 " "

Prob. 7. What e.m.f. in volts is produced when the conductor shown in Fig. 20 cuts across the uniform magnetic

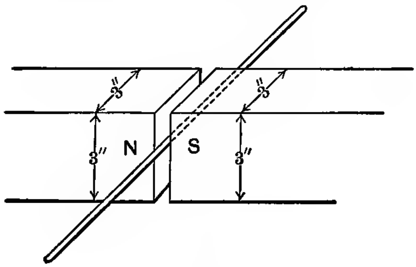


FIG. 20.

field at a uniform velocity of 5000 ft. per minute? There are 1,000,000 maxwells of magnetism in the field. ($\Phi = 1,000,000.$)

3.33 volts. *Ans.*

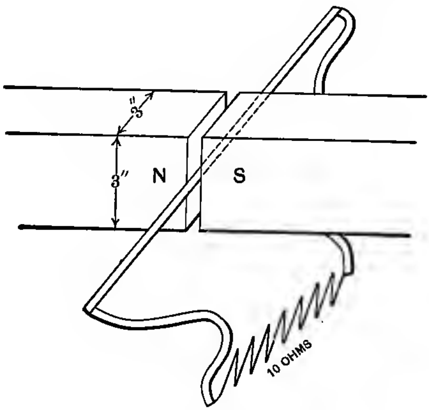


FIG. 21.

Prob. 8. The resistance of the circuit in Fig. 21 is 10 ohms. If a part of the circuit passes at constant velocity

through a uniform field 3 in. square in which the total magnetism is 725,000 maxwells: (*a*) What current will flow through the wire when the circuit cuts the field in .01 second? (*b*) In .005 second? (*c*) How many joules of energy will be lost in heating the wire in (*a*)? (*d*) In (*b*)?

(*a*) .0725 ampere. *Ans.*
(*b*) .1450 " "
(*c*) .0005256 joule. "
(*d*) .0010512 " "

Prob. 9. A battery furnishing a pressure of 10 volts is in a circuit of 2 ohms resistance. A part of the circuit is in a uniform field 3 in. square, in which the magnetism is 1,000,000

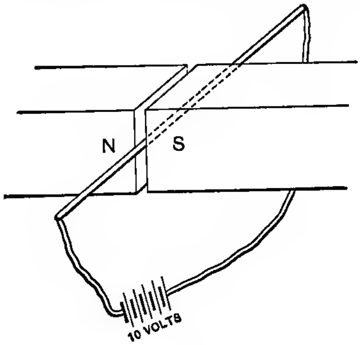


FIG. 22.

maxwells. With what force, measured in pounds, will the wire be urged across the field? (See Fig. 22.)

.1474 lb. *Ans.*

Prob. 10. The e.m.f. at the terminals of the condenser shown in Fig. 23 is changed from 1000 volts positive to 1000 volts negative in $\frac{1}{60}$ second. The capacity of the condenser is 7 microfarads. (*a*) What average current flows for the time? (*b*) How much energy is stored in the condenser at 1000 volts pressure?

(*a*) .84 ampere. *Ans.*
(*b*) 3.5 joules. "

Prob. 11. The current in the circuit shown in Fig. 24 is changed from 3 to 10 amperes in .2 second. If .01 volt opposes the change, what is the value of the inductance in henrys? .000285 henry. *Ans.*

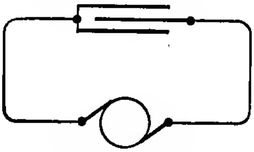


FIG. 23.

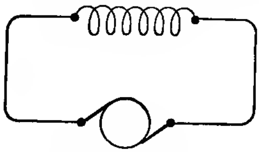


FIG. 24.

Prob. 12. The current in a circuit similar to that of Fig. 24 rises from 10 to 20 amperes in two seconds. The inductance is 3 henrys. What average volts are consumed if the resistance of the circuit is three ohms (*a*) while current is rising? (*b*) When current has become uniform?

(*a*) 60 volts. *Ans.*
(*b*) 60 " "

CHAPTER III.

PERIODIC CURVES.

SYNOPSIS.

10. Properties of the Sine Curve.—*a*. An alternating quantity is a quantity the values of which are alternately positive and negative. The successive values of such a quantity may follow a simple law, or one which is more or less complex. The simplest possible law is

$$y = A \sin x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In this equation x is a continually increasing angle, and y is, therefore, alternately positive and negative. Its maximum value in either direction is A, which occurs when x is 90°, or 90° plus any multiple of 180°.

A plane curve drawn in any system of co-ordinates and giving graphically the relation between x and y in the above equation is called a **curve of sines,** or often in electrical literature a **sine curve.**

b. The Sine Curve in Rectangular Co-ordinates.—If successive values of the angle x are plotted as abscissæ, and the corresponding values of y are taken as ordinates, the resulting sine curve is of the form shown in Fig. 25. A convenient

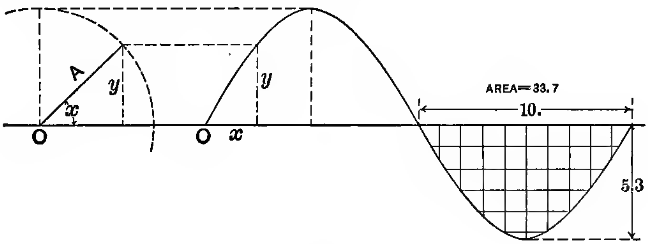


FIG. 25.

method of plotting the curve is there given. If A be a constant radius vector which starts from a horizontal position and rotates in a positive direction, counter-clockwise, its projection on a vertical line at any instant is equal to A sin x, where x is the angle between A and the origin.

c. Time as Abscissæ for Sine Curves.—Sine curves will be used in this text to represent graphically alternating quantities which vary with time. It is often convenient, therefore, to use time as the independent variable in the equation

$$y = A \sin x.$$

This may be done by expressing x in terms of time. As x is a uniformly increasing angle, it varies directly as time, and its

value at any instant is determined by the time which has elapsed since the radius vector left the initial position, and by the angular velocity of the radius vector. Thus when ω equals the angular velocity of the radius vector, its angular position with respect to its initial position is, at the time t,

$$x = \omega t.$$

Substituting in (2),

$$y = A \sin \omega t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the equation of the sine curve in terms of angular velocity and time. If T be the time of a complete revolution of the radius vector,

$$T = \frac{2\pi}{\omega},$$

or

$$\omega = \frac{2\pi}{T}.$$

By substitution, equation (3) then becomes

$$y = A \sin \frac{2\pi}{T} t. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

d. The Sine Curve in Polar Co-ordinates.—When the same values of x and y, as determined in the preceding section, are plotted in polar co-ordinates, the resulting curve is a circle, Fig. 26. The negative sine values are found for values of x between 180° and 360°. In this part of the cycle the curve is traced over by the negative end of the radius vector.

e. The Average Value of a Sine Curve.—By "value of a curve of sines" is meant the value of an ordinate to the curve as drawn in rectangular co-ordinates; i.e., a y value. The average value for a complete period, corresponding to a complete revolution of the vector A, Fig. 25, is zero. The average value for a half period is obtained by dividing the area of a loop of the curve by its length, i.e., by π when the angles

are measured in radians. For an actual curve, in rectangular co-ordinates, this area may be obtained by means of the planimeter, or, if plotted on cross-section paper, by counting

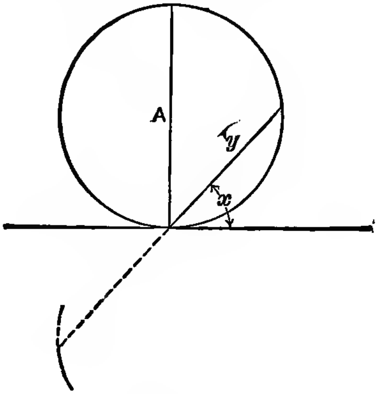


FIG. 26.

the enclosed squares. Thus, in Fig. 25, there are 33.7 squares enclosed by one loop of the curve, while the length of the loop is 10. The average ordinate is, therefore, 33.7 divided by 10, or 3.37. This corresponds exactly with the value found analytically, which is:

$$\text{Average ordinate} = \frac{2}{\pi} \times \text{maximum ordinate}$$

$$= \frac{2}{\pi} \cdot 5.3 = 3.37.$$

Analytically the same result is obtained by a simple integration, as follows:

$$y = A \sin x,$$

$$y\,dx = A \sin x\,dx,$$

$$\text{Area}_0^\pi = \int_0^\pi y\,dx = A\int_0^\pi \sin x\,dx$$

$$= A\Big[-\cos x\Big]_0^\pi = 2A,$$

and average ordinate $= \dfrac{area}{base} = \dfrac{2A}{\pi}.$

The **average value** *of a sine curve is* $\dfrac{2}{\pi}$ *times its maximum value.*

f. The Effective Value of a Sine Curve.—By "effective value" is meant the *square root of the mean of the squares* of the instantaneous values. This is always positive and has the same value whether taken for an entire period or for a half period. In Fig. 27 is shown the sine curve b and the curve

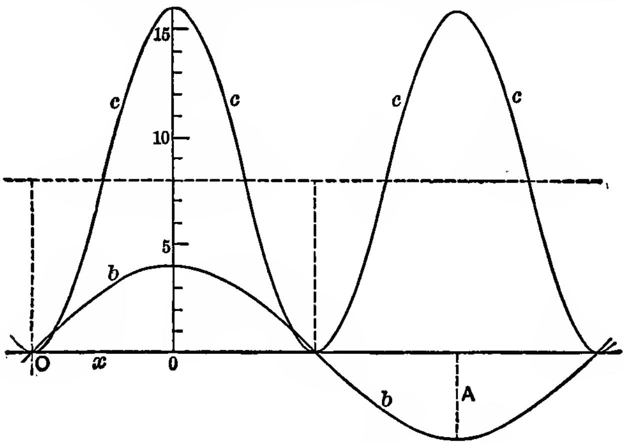


FIG. 27.

of squares c. Each ordinate of the latter curve is the square of the corresponding ordinate of the former. The average value of c may be obtained by the methods used for determining the average value of the sine curve. The **effective value** of $b = \sqrt{average\ c}$.

1. *Analytical Determination from the Rectangular Curve.*

For curve b, $y = A \sin x$.

" " c, $y = A^2 \sin^2 x$. . . . . (5)

$$\textit{Area of loop of } c = \int_0^\pi y\,dx = A^2\int_0^\pi \sin^2 x\,dx$$

$$= \frac{A^2}{2}\int_0^\pi (1 - \cos 2x)dx$$

$$= \frac{A^2}{2}\left[\int_0^\pi dx - \int_0^\pi \cos 2x\,dx\right]$$

$$= \frac{A^2\pi}{2}.$$

$$\textit{Average value of } c = \frac{A^2}{2}.$$

$$\textit{Effective value of } b = \frac{A}{\sqrt{2}}.$$

2. *Graphical Determination from the Polar Curve.*—It has been pointed out that a sine curve, when plotted in polar co-ordinates, is a circle. The area of the circle may be considered as made up of infinitesimal triangles, with apexes at the centre of revolution of the vector and having an angular width of $\Delta\alpha$ and altitude b, as in Fig. 28. The areas of the

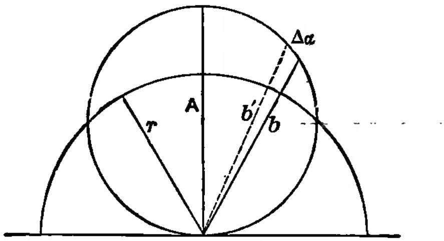


FIG. 28.

triangles will be proportional to the squares of their altitudes, as the angle $\Delta\alpha$ is constant, and therefore the total area is proportional to the mean of the squares of the altitudes of the triangles. The complete circle is generated while the radius vector, b, sweeps through an angle of 180°.

If a semicircle be drawn, as shown in Fig. 28, with the diameter tangent to the circle, its area will be the same as that

of the circle, if the square of its radius, r, is equal to the mean of the squares of b. This is evident from the facts that the area of the semicircle may be considered as made up of triangles as in the preceding case and therefore proportional to r^2, and that the radius, r, sweeps through the same angle in generating the semicircle as was swept through by b in generating the circle. *That is, the average of the squared radius vectors of the sine curve in polar co-ordinates is the square of a radius vector of constant length which would sweep over the same area in passing through an angular distance of 180°.*

As the areas of the circle and the semicircle are equal,

$$\frac{\pi r^2}{2} = \frac{\pi A^2}{4},$$

$$r = \frac{A}{\sqrt{2}},$$

where A is the diameter of the circle. But $r^2 =$ *the mean of* b^2, and

$$r = \sqrt{\textit{mean of } b^2} = \textit{effective } b = \frac{A}{\sqrt{2}}.$$

11. **Combinations of Sine Curves.**—*a. Phase Relations of Sine Curves.*—A more general form for the equation of a sine curve is

$$y = A \sin (x + \alpha), \quad \ldots \ldots \quad (6)$$

where α is a constant angle, and x is an angle increasing from 0° at a uniform rate. The value of the ordinate to the curve on starting, or at the time zero, is

$$y_0 = A \sin \alpha.$$

The curve

$$y' = A \sin (x + \alpha)$$

and its generating radius vector, A, are drawn in Fig. 29.

A second curve, b, is drawn in the same figure. The equation of the second curve is

$$y'' = B \sin (x + \beta),$$

where β is a constant angle.

The two curves are said to **differ in phase position** by the angle $(\beta - \alpha)$, as that is the angular difference between corresponding points on a and b, say between the points at which

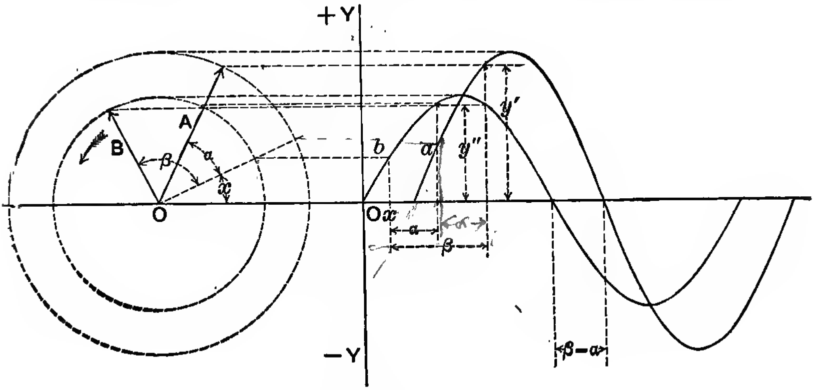


FIG. 29.

they cross the X axis in the same direction. The radius vectors A and B also differ in phase position by the angle $(\beta - \alpha)$. Since counter-clockwise rotation is taken as positive, the radius vector B is ahead of A in angular position, and the sine curve b is ahead of the sine curve a, or it is said to **lead** a by an angle of $(\beta - \alpha)^\circ$. Conversely, a is said to **lag** an angle of $(\beta - \alpha)^\circ$ behind b.

When the angle between two sine curves is 90°, the curves are said to be **in quadrature** (Fig. 30).

When $\beta = \alpha$, the radius vectors coincide, and the sine curves are said to be **in phase** see (Fig. 31). Their equations are then

$$y' = A \sin (x + \alpha),$$
$$y'' = B \sin (x + \alpha),$$

and the ratio between ordinates to the two curves, drawn through the same point, x, is

$$\frac{y'}{y''} = \frac{A \sin (x + \alpha)}{B \sin (x + \alpha)} = \frac{A}{B} = a \ constant.$$

One curve may therefore be derived from the other by multiplying each ordinate of that other by a constant.

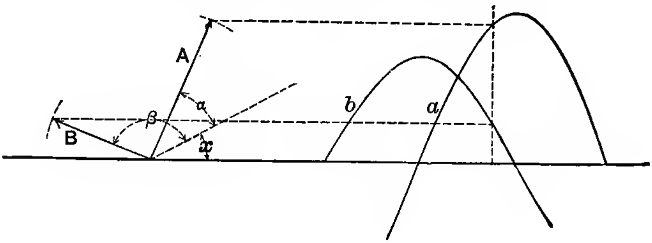


FIG. 30.

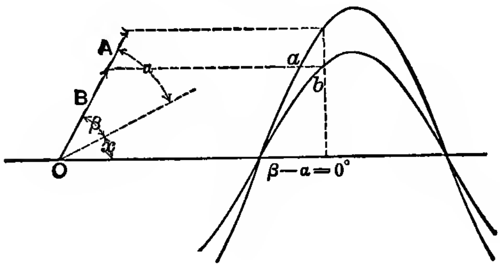


FIG. 31.

The ratio between corresponding ordinates of two sine curves in phase is constant.

b. Addition of Sine Curves of the Same Frequency.—Sine curves are said to have the same **frequency** when their generating radius vectors revolve with the same angular velocity.

Sine curves may be added by the addition of corresponding ordinates, i.e., ordinates lying in the same vertical line. Each such addition gives the corresponding ordinate to the curve of

sums. The curve of sums may be plotted by the determination of a large number of such points.

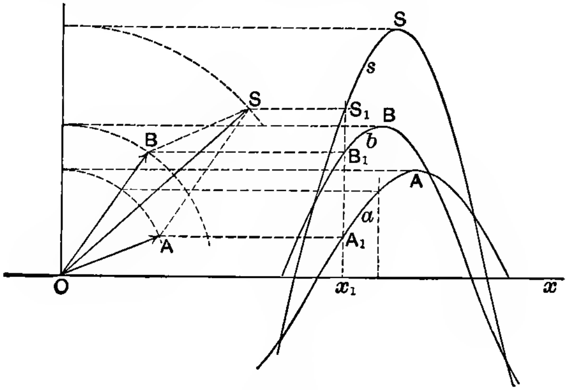


FIG. 32.

In Fig. 32, the sine curves a and b have the same frequency, and are generated by the radius vectors A and B. It is required to find the form of the curve s, which is their sum. Construct the parallelogram $OASB$.

The ordinate to the sine curve a is at any instant, x_1,

$$OA \sin xOA = A_1x_1.$$

The ordinate to the sine curve b is, at the same instant,

$$OB \sin xOB = B_1x_1.$$

The ordinate to the curve of sums, s, is, at the same instant,

$$S_1x_1 = A_1x_1 + B_1x_1,$$

where

$$S_1A_1 = B_1x_1,$$

$$AS \sin A_1AS = OB \sin xOB = B_1x_1 = S_1A_1,$$

and

$$OS \sin xOS = OA \sin xOA + AS \sin A_1AS$$
$$= A_1x_1 + B_1x_1 = S_1x_1.$$

Therefore S_1x_1 is the projection of OS on the ordinate through x_1.

Since this is true for any position of the parallelogram $OASB$, it follows that

The sum of two sine curves having the same frequency is the sine curve which is generated by the diagonal of the parallelogram formed on the radius vectors of the component curves.

c. Sum of Sine Curves in Quadrature.—The equation

$$y'' = B \cos x$$

may be written

$$y'' = B \sin \left(x + \frac{\pi}{2}\right)$$

when it is seen to be a sine curve that is 90° ahead of the curve

$$y' = A \sin x.$$

The sum of the curves

$$y' = A \sin x \quad \text{and} \quad y'' = B \cos x$$

is therefore a sine curve which is generated by a radius vector forming the diagonal of the rectangle produced by drawing A, making the angle x, and B, making the angle $x + \frac{\pi}{2}$, with

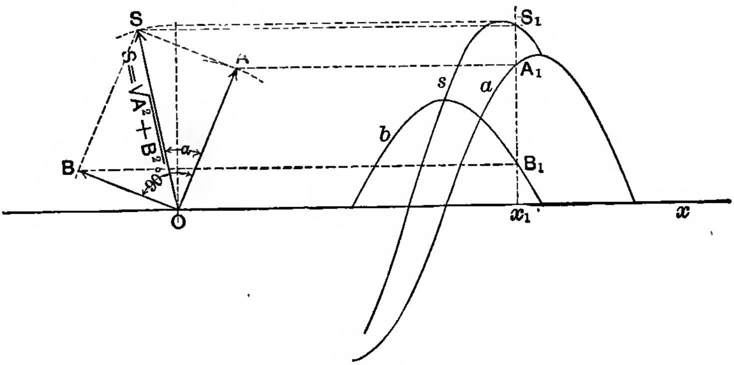


FIG. 33.

the zero position. (See Fig. 33.) The value of this diagonal must be

$$S = \sqrt{A^2 + B^2}.$$

The magnitude of the sum is dependent on the magnitude of A and B, while its phase position, or the angle α which it makes with A, is governed by the ratio between A and B. Thus

$$\frac{B}{A} = \tan \alpha, \quad \text{or} \quad \alpha = \tan^{-1} \frac{B}{A}.$$

α may therefore vary between the limits 0° and 90°. If x be zero, and A and B are positive, S lies in the first quadrant.

With A negative	and B positive,	S lies	in the	second	quadrant.
" A "	" B negative,	S "	"	third	"
" A positive	" B "	S "	"	fourth	"

(See Fig. 34.)

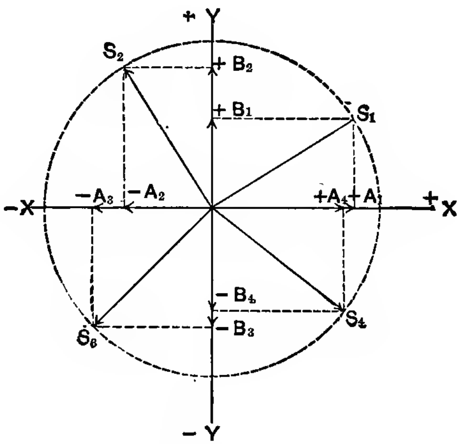


FIG. 34.

It follows, therefore, that by properly choosing the factors A and B, the sum of two curves,

$$y' = A \sin x \quad \text{and} \quad y'' = B \cos x,$$

may be made a sine curve having any amplitude, and having any phase position. It is frequently more convenient to

express a sine curve as the sum of the corresponding sine and cosine curves, thus:

$$y = A \sin x + B \cos x, \quad . \quad . \quad . \quad . \quad (7)$$

instead of

$$y = S \sin(x + \alpha).$$

√ *d. Product of Sine Curves of the Same Frequency.*—By product of sine curves is meant the curves of products of instantaneous values.

1. *When the sine curves are in phase.* Let it be required to find the product of the sine curves b and b', Fig. 35.

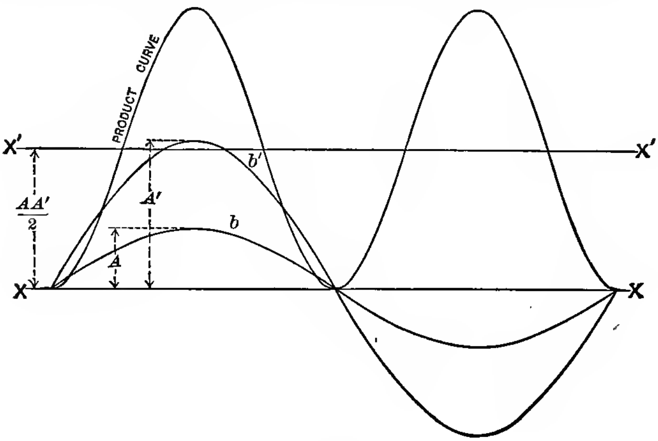


FIG. 35.

Their equations are

$$y = A \sin x,$$
$$y' = A' \sin x,$$

and the product of their ordinates at any instant is

$$yy' = y_1 = AA' \sin^2 x. \quad . \quad . \quad . \quad . \quad (8)$$

Expanding this, it becomes

$$y_1 = \frac{AA'}{2}(1 - \cos 2x). \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Transposing the axis of this curve from XX to $X'X'$ a distance of $\frac{AA'}{2}$, its equation becomes

$$y_2 = y_1 - \frac{AA'}{2} = -\frac{AA'}{2}\cos 2x, \quad . \quad . \quad (10)$$

$$y_2 = -\frac{AA'}{2}\sin\left(2x + \frac{\pi}{2}\right), \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which is a sine curve of twice the frequency of b or b', with its axis at a distance $\frac{AA'}{2}$ above the axis of b and b'. The average value of this curve referred to the axis XX is $\frac{AA'}{2}$. This may be written $\frac{A}{\sqrt{2}}\cdot\frac{A'}{\sqrt{2}}$, when it is seen to be the product of the effective values of b and b'.

The average value of the product of two sine values in phase is the product of their effective values. This is one-half the product of their maximum values.

2. *When the sine curves are in quadrature.* Let it be required to find the product of the sine values b and b', Fig. 36, which are in quadrature.

Their equations are

$$y = A \sin x,$$

$$y' = A' \sin\left(x + \frac{\pi}{2}\right) = A' \cos x,$$

$$yy' = (A \sin x)(A' \cos x) = \frac{AA'}{2}\sin 2x. \quad . \quad (12)$$

This is a sine curve of double frequency, the axis of which coincides with the axis of b and b'. Its average value taken for a complete period is zero.

The average value of the product of two sine curves in quadrature is zero.

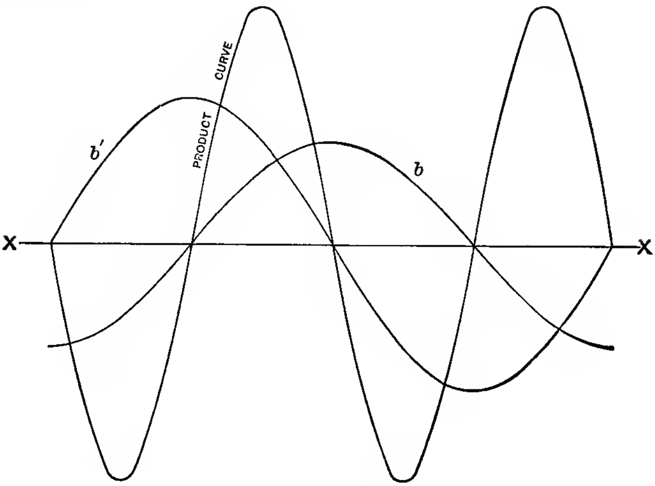


FIG. 36.

3. *When the sine curves are not in phase and not in quadrature.* Let it be required to find the product of the sine curves b and b', Fig. 37. One of these sine curves, say b', may be divided into components, one of which is in phase, and the other in quadrature, with b. These components are generated by the radius vectors

$$OB_1 = OB \cos \theta,$$
$$OB_2 = OB \sin \theta,$$

where θ is the angle of phase difference between A and B. The product of A and B is the sum of the products of A with each of the components of B. These products are given by (11) and (12). The average quadrature product is zero. The maximum value of the component of B which is in phase with A is

$$OB_1 = OB \cos \theta.$$

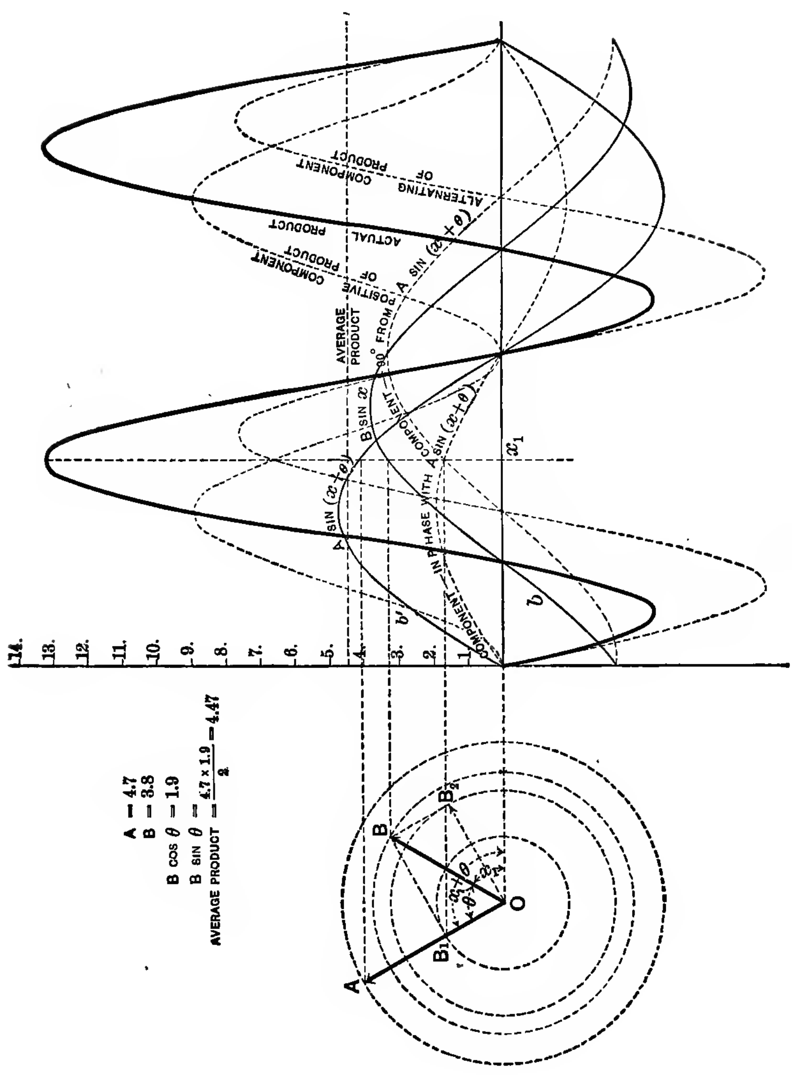

A = 4.7
B = 3.8
B cos θ = 1.9
B sin θ =
AVERAGE PRODUCT = 4.7 x 1.9 / 2 = 4.47
AVERAGE PRODUCT
ACTUAL PRODUCT
COMPONENT OF POSITIVE PRODUCT
ALTERNATING COMPONENT OF PRODUCT
A SIN (x+θ)
B SIN x
COMPONENT IN PHASE WITH A SIN (x+θ)
COMPONENT 90° FROM A SIN (x+θ)
O
A
B
B1
B2

FIG. 37.

The average product of the sine values A and B is therefore

$$\frac{OA}{\sqrt{2}} \cdot \frac{OB}{\sqrt{2}} \cdot \cos \theta.$$

The average value of the product of two sine curves is the product of their effective values, times the cosine of their angle of phase difference.

e. The Rate of Change Curve of Sine Values.—The rate at which the ordinates to a sine curve change their values is variable, and if plotted in rectangular co-ordinates produces a

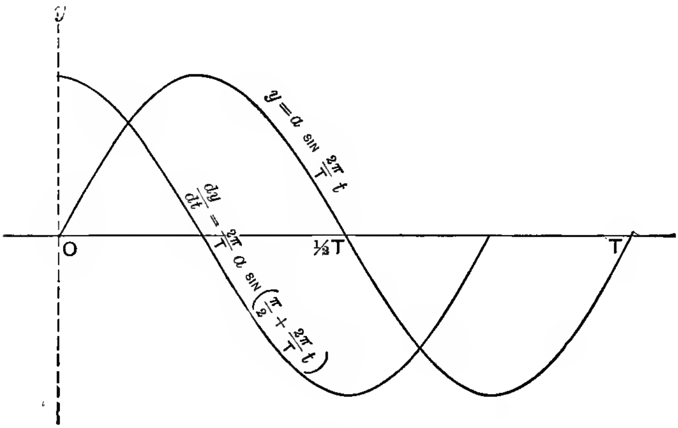


FIG. 38.

sine curve in quadrature with the one from which it is derived. (See Fig. 38.) The first curve is

$$y = a \sin x,$$

or plotted with time as abscissæ it is

$$y = a \sin \frac{2\pi}{T} t,$$

where T is the time of one complete period.

Then

$$dy = \frac{2\pi}{T} a \cos \frac{2\pi}{T} t \, dt,$$

and the rate of change

$$\frac{dy}{dt} = \frac{2\pi}{T} a \sin\left(\frac{\pi}{2} + \frac{2\pi}{T} t\right). \quad . \quad . \quad . \quad . \quad (13)$$

The rate of change curve is therefore a sine curve 90° in advance of the original curve, and its values are $\frac{2\pi}{T}$ *times as great. Conversely: The integral of a sine curve is a second sine curve 90° behind the original curve, and its values are* $\frac{T}{2\pi}$ *times as great.*

f. The Addition of Sine Curves of Different Frequencies.— The curve resulting from the addition of sine curves of different frequencies is non-sine in form. It is always irregular; positive and negative portions taken with reference to the X axis may or may not be alike.

The investigation of this fact led to the discovery by Fourier that periodic sine curves of different frequencies may always be found in value, phase positions, and frequencies such that their sum will produce a periodic alternating curve of any desired form.

Fourier determined a systematic method for obtaining the periodic sine curves of different frequencies which when added would reproduce any given periodic curve however irregular it might be. Such a series is known as a **Fourier's Series.**

12. The Fourier's Series for Any Periodic Alternating Curve.—If an irregular curve be plotted in rectangular co-ordinates and the length of a complete period is made equal to 2π, or 360°, it may be reconstructed by the addition of sine waves properly selected as to amplitude and phase position, one of which has the same frequency, or wave length, as the irregular wave, and others having twice, three times, four times, . . . n times that frequency, and having wave lengths of 1/2, 1/3, 1/4, . . . $1/n$ that of the irregular wave.

The component sine curves which combine to produce the total curve are known as the first, second, third, . . . etc. **harmonics.** The first harmonic, or **fundamental,** has the frequency of the total curve; the second harmonic twice that frequency, and so on. This convention differs slightly from that adopted in music, where a tone having twice the frequency of the fundamental is called the first harmonic. The change in convention is used for convenience only.

The general equation for a curve made up of harmonic sine components is

$$y = A' \sin (x + \alpha_1) + A'' \sin (2x + \alpha_2) \\ + A''' \sin (3x + \alpha_3) + \dots + A^n \sin (nx + \alpha_n). \quad (14)$$

A_1, A_2, A_3, . . . A_n are the amplitudes of the component sine waves, and α_1, α_2, α_3, . . . α_n are constant angles which specify the position of the radius vectors generating the component curves when x is zero or is any multiple of 360°. These relations are shown in Fig. 39.

In this figure the irregular curve S is made up of three harmonic sine components, having frequencies of once, twice, and three times that of the irregular curve. The radius vectors A', A'', and A''' which generate the component curves rotate with different angular velocities, and are numbered 1, 2, and 3 to correspond to their velocities. In the figure these radius vectors are drawn in heavy lines for the position in which x is zero, and in dotted lines for the position $x = x_1 = 45°$. The equations of the fundamental, second, and third harmonic sine components of S are

$$y' = A' \sin (x + 30°),$$

$$y'' = A'' \sin (2x + 60°),$$

$$y''' = A''' \sin (3x + 120°),$$

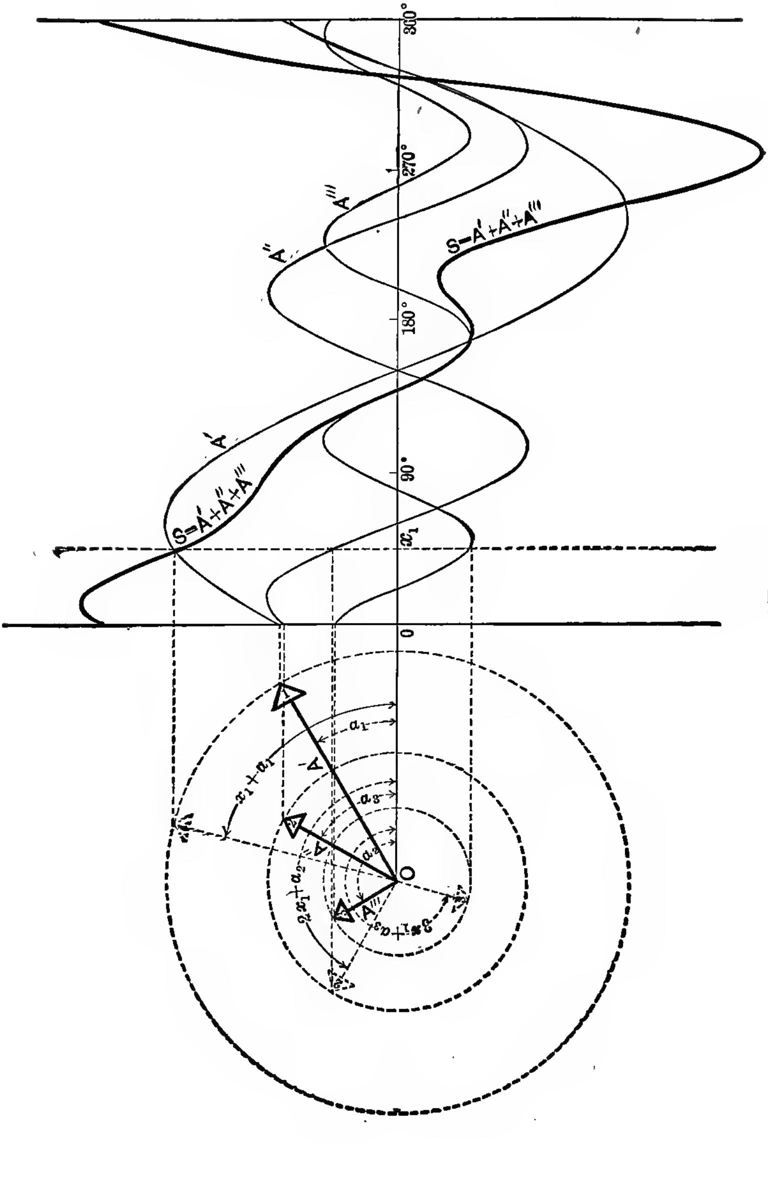


FIG. 39.

and the equation of the curve S is

$$y = A' \sin (x + 30°) + A'' \sin (2x + 60°) \\ + A''' \sin (3x + 120°). \quad (15)$$

Note that this curve does not have like loops above and below the X axis.

From the statements already made it follows that by properly selecting the values A', A'', A''', etc., and the constant angles of phase displacement; α_1, α_2, α_3, etc., the above equation may represent any *finite*, *continuous*, and *periodic* curve whatever, no matter how irregular it may be.

13. Analysis of a General Periodic Curve.—The separation of an irregular periodic curve into its components is always possible, and usually it will be found that the first five components give a sufficiently close approximation to the total curve to answer all practical purposes. The actual analysis may be made graphically or analytically. In either case the method is based on the following laws:

1. *The average of the product of any harmonic with any other harmonic is zero when taken for a complete cycle of the irregular curve.*

2. *The average of the product of two sine curves in phase, and having the same frequency, is half the product of their amplitudes.*

3. *The average of the product of two sine curves in quadrature and having the same frequency is zero.*

The discussion of the first law is beyond the scope of the present text.* The meaning of the law may be illustrated graphically as in Figs. 40 and 41. In Fig. 40 the sine curve b has twice the frequency of the fundamental curve a. Their product taken over a complete period of a gives a curve having loops of equal areas above and below the X axis. The

* See "Fourier's Series and Spherical Harmonics"—Byerly.

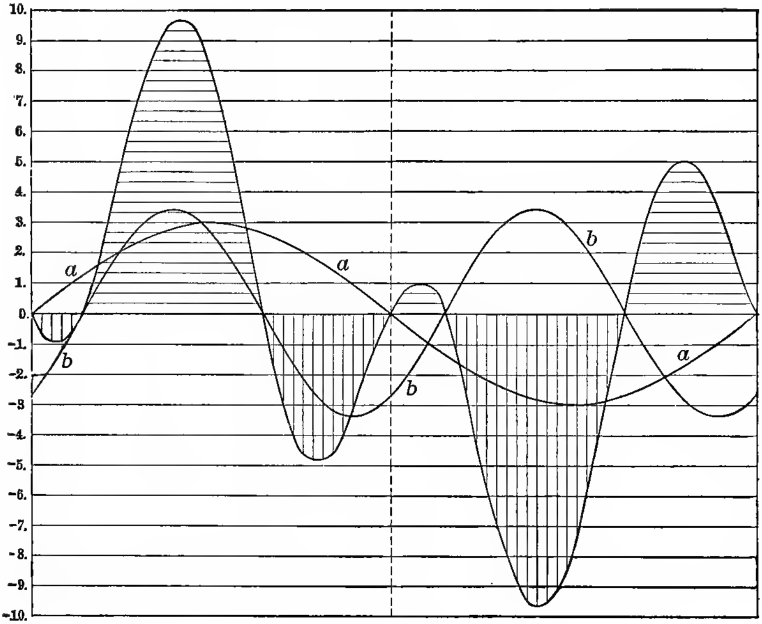


FIG. 40.

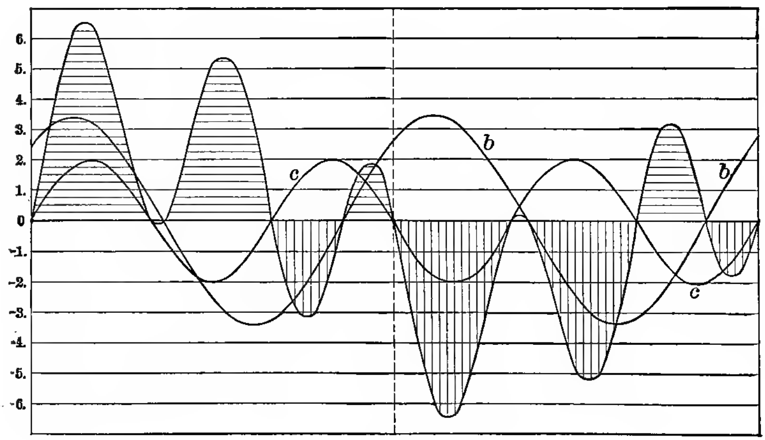


FIG. 41.

average ordinate of the product is therefore zero. Similarly in Fig. 41 the product of b, having twice the frequency of a, and c, having three times the frequency of a, gives a product curve which has the average ordinate zero.

For the proof of the second law see Section 11, d, 1, page 38, and for the third law see Section 11, d, 2, page 39.

The product of one quantity by another is identical with the sum of the products of that quantity and each of the components of the other. Thus,

$$x(a + b) = xa + xb. \quad . \quad . \quad . \quad . \quad (16)$$

It follows, therefore, that the average ordinate, taken over a complete period, of the product of an irregular curve and a sine curve having n times its frequency, is equal to the average ordinate of the product of this sine curve and the component of the nth harmonic of the irregular curve which is in phase with it. The average ordinate, taken between the same limits, of the product of this sine curve and any other harmonic component of the total curve, or the sum of all the other harmonic components, is zero.

The general equation

$$\begin{aligned} y = {} & A' \sin (x + \alpha_1) + A'' \sin (2x + \alpha_2) \\ & + A''' \sin (3x + \alpha_3) + \ldots + A^n \sin (nx + \alpha_n \end{aligned}$$

may be rewritten by Section 11, c, p. 37, eq. (7), as follows:

$$\left.\begin{aligned} y = A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \ldots \\ + A_n \sin nx, \\ + B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \ldots \\ + B_n \cos nx. \end{aligned}\right\} \quad (17)$$

Let it be required to find any coefficient, say A_3, and the phase angle, α_3, of this equation, when the curve itself is given.

From Section 11, pages 35 and 36,

$$(A''')^2 = A_3^2 + B_3^2,$$

and

$$\alpha_3 = \tan^{-1}\frac{B_3}{A_3}.$$

Multiply graphically a complete wave of the total curve by the curve

$$y_3' = \sin 3x.$$

The average ordinate of this product, E, is equal to the average ordinate of the product of the latter factor by the curve $y_3 = A_3 \sin 3x$, since the product of $y_3' = \sin 3x$ with each of the other components of y is zero. But the average ordinate of the product of

$$y_3' = \sin 3x$$

and

$$y_3 = A_3 \sin 3x$$

is

$$\frac{A_3 \times 1}{2},$$

and this may be placed equal to the average ordinate of the graphic multiplication, or

$$A_3 = 2E.$$

Similarly, as F is the average ordinate of the product of the total curve and the curve

$$y_3'' = \cos 3x,$$

it follows that $B_3 = 2F$. The sine and cosine components of this harmonic, A''', have thus been determined. It was shown above that the amount and phase position of this harmonic are, therefore, immediately known.

The analysis of a general curve may be often much simplified by an inspection of its form. If only odd harmonics, i.e., the first, third, fifth, etc., are present, the phase relations between components will be similar at the end of each half

wave-length of the fundamental. (See Fig. 45.) Such a curve will have like loops above and below the X axis, and, conversely,

Curves having like loops above and below the X axis contain only the odd harmonics.

This is the case with nearly all curves which represent the behavior of electrical machinery. In this case the average ordinate of the graphical multiplications may be taken over a half -wave instead of over a complete wave, since the product curve repeats itself every half period of the fundamental, where only odd harmonics are present.

Where the even harmonics are present, i.e., the second, fourth, sixth, etc., the phase positions of the components are similar only at the close of each complete period of the general periodic curve. The result is that all such general curves must be unsymmetrical about their axes; the plus and minus portions of each period must differ in form.

Curves having unlike loops above and below the X axis in general contain both odd and even harmonics.

The following problem is given to illustrate fully the application of the above method of analysis by which any periodic alternating curve may be resolved into its harmonic components.

Problem. It is required to determine the harmonic components of the irregular curve drawn with the heavy line in Fig. 42.

This curve is symmetrical about its axis and should contain no even harmonics. This fact is tested in the diagram of Fig. 42, in the case of the second harmonic.

The unit sine curve of double frequency drawn in this diagram constitutes the **analyzer** for the sine component of the second harmonic. It is drawn to a larger scale than the original curve by the ratio 40 : 1. The original curve is multi-

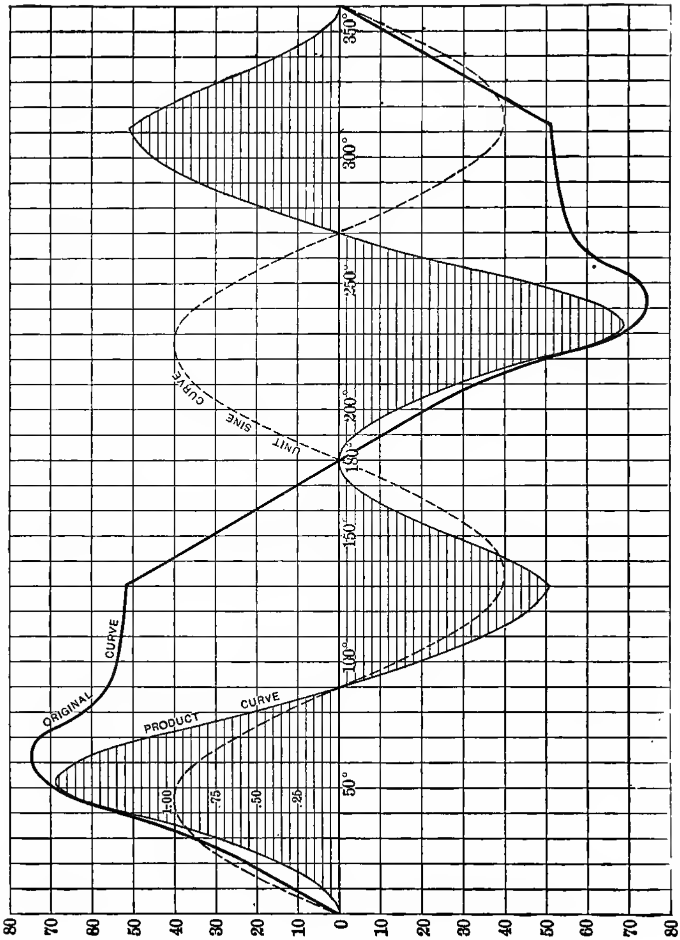


FIG. 42.—Harmonic Analysis. Sine Product of Second Harmonic and Irregular Curve. (Average of product is zero.)

plied by this analyzer, the result being the product curve as laid down in the diagram.

An inspection of this product curve shows that its average value must be zero. From symmetry the positive and negative areas for a complete period that the curve encloses with the axis must be equal, their sum must be zero, and the average value of the product curve must, therefore, be zero. This result is due evidently to the fact that the relative phase positions of the second harmonic with reference to the plus and minus portions of the original curve are such as to give alternately plus and minus signs to corresponding product values. The relative phase positions of the original curve and the second harmonic sine analyzer are such as to give product values throughout a complete cycle symmetrically located with reference to the axis. For every plus value there is a corresponding negative value causing the average value of the product curve to be zero.

A trial of a cosine second harmonic analyzer will produce the same result, and for the same reasons.

Further trial of any even harmonic analyzer will produce an average zero product for similar reasons.

It follows from this test, therefore, that this symmetrical irregular curve contains no even harmonics, otherwise the product of it and some of the even harmonic analyzers would give positive average values.

The curve being quite irregular and symmetrical will be found to be rich in the odd harmonics. In Figs. 43 and 44 are given diagrams showing the application of the sine and cosine third harmonic analyzers to determine the value and phase position of the third harmonic of the original curve. In Fig. 43 the unit sine analyzer for the third harmonic analyzer is drawn and the product curve determined as shown. The

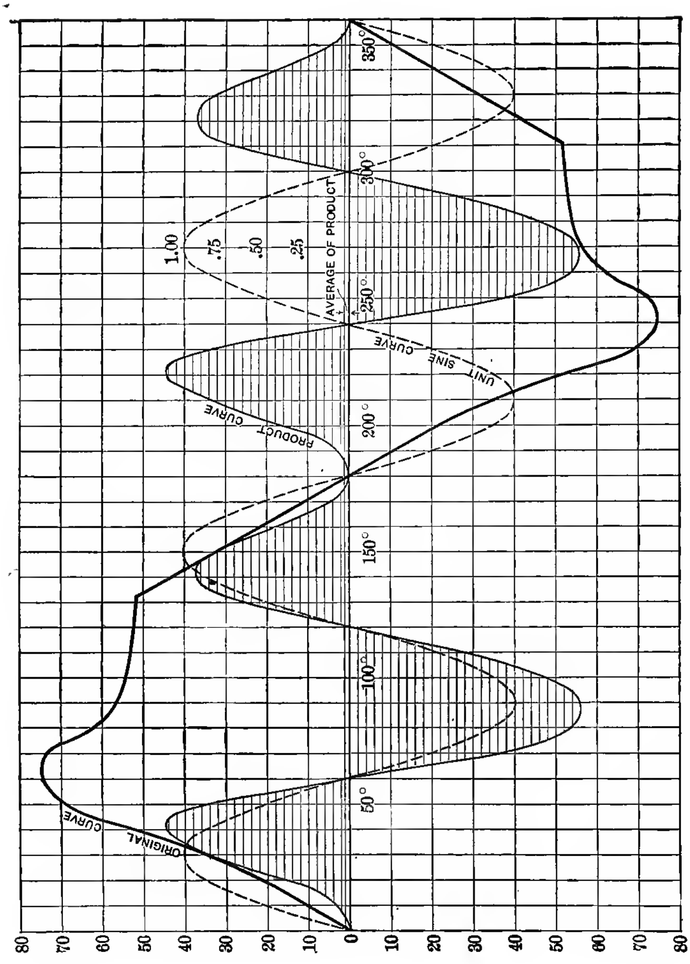


FIG. 43.—Harmonic Analysis. Sine Product of Third Harmonic and Irregular Curve.

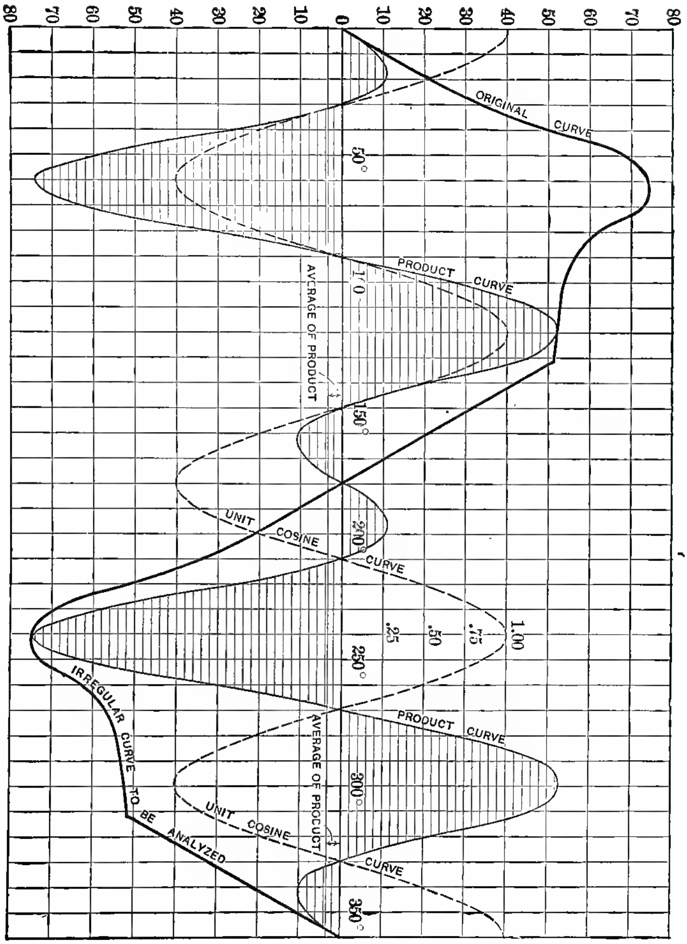


FIG. 44.—Cosine Product of Third Harmonic and Irregular Curve.

positive and negative areas of the product curve are then determined by means of a planimeter. The final net area is divided by the length taken to actual scale of a complete period, producing thereby the average value of the product. This value is drawn in Fig. 43, and so labeled. In doing this the numerical work is as follows:

Positive area	5140
Negative area	4546
Net positive area	594

Length of one period 360

Average product $\frac{594}{360} = +1.65$

The maximum value of the sine component of the third harmonic of the original curve is therefore

$$A_3 = 2(+1.65) = +3.30.$$

Fig. 44 gives graphically a corresponding analysis to determine the cosine component of the third harmonic of the original curve. The numerical work for this is as follows:

Positive area	4334
Negative area	5562
Net negative area	1228

Length of one complete period 360

Average product $\frac{-1228}{360} = -3.41$

$$B_3 = 2(-3.41) = -6.82.$$

The total third harmonic component is therefore

$$A''' = \sqrt{A_3^2 + B_3^2},$$

$$A''' = \sqrt{3.3^2 - 6.82^2}, \qquad + \tan^{-1} \frac{-6.82}{3.3},^*$$

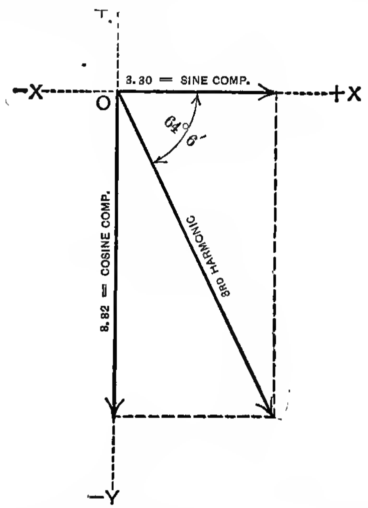
$$A''' = 7.56, \quad -64° \, 6'.$$

This third harmonic component is drawn to scale in amount and phase position in Fig. 45.

The fundamental and the fifth harmonics have been determined by this method. The only difference in detail met with consists in the use of sine and cosine unit analyzers having periodicities corresponding to those of the first and fifth harmonics which they are to determine. The numerical values and phase positions of these harmonics are given in the following equation:

$$y = 66.8 \sin x + 4.4 \cos x + 3.3 \sin 3x - 6.82 \cos 3x - 7.24 \sin 5x + 1.99 \cos 5x. \quad (18)$$

* The accompanying figure will assist the reader to see that the ratio of the



cosine to the sine components gives the tangent of the angle of phase difference, and not the cotangent as might at first be supposed.

The equation may conveniently and obviously be written in the following manner:

$$y = (66.8_1 + 3.3_3 - 7.24_5) + j(4.4_1 - 6.82_3 + 1.99_5),*$$

or, by combining the quadrature components of the harmonics, it may be written:

$$y = 66.9 \sin(x + 3^\circ 54') + 7.56 \sin(3x - 64^\circ 6') + 7.5 \sin(5x + 164^\circ 36').$$

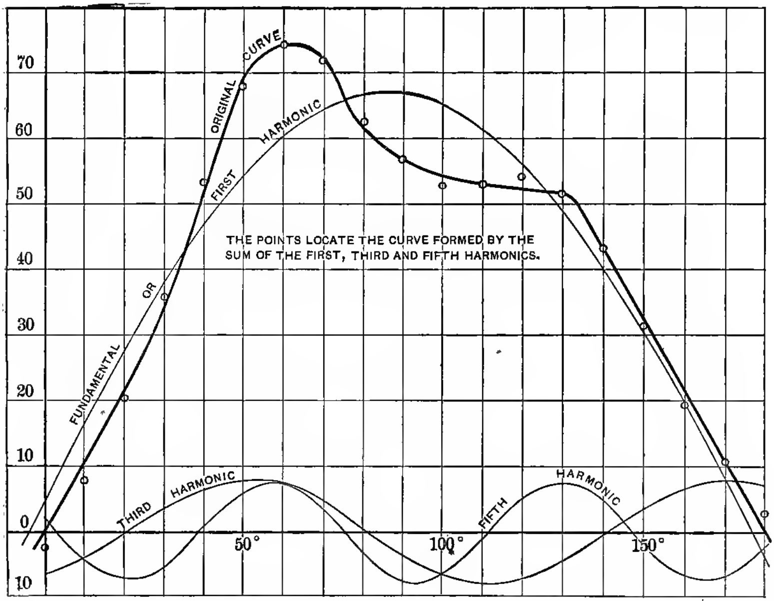


FIG. 45.—Harmonic Analysis showing Original Curve, its Component Sine Curves, and the Curve due to their Combination.

In Fig. 45 the first and fifth harmonics have also been drawn in their respective amounts and phase positions. Finally, at intervals of 10° their sum has been taken. Points corresponding to these sums have been plotted. It is seen that they come so near in most cases to the original curve that it was thought best for the sake of clearness not to draw in the complete curve given by the sum of the first, third, and fifth

* j indicates the sine and cosine relation of these quantities.

harmonics. Evidently if the original curve had more pronounced minor irregularities there would be more of a residual difference between it and the summation curve in Fig. 45. Such residual could then be expressed by the seventh, ninth, etc., harmonics.

By means of the method illustrated in the above problem any periodic curve may be broken up into its harmonic components. Conversely, any periodic curve may be expressed as the sum of its harmonic components.

All problems that arise in the treatment of general periodic or general alternating quantities may be solved by the treatment of their harmonic components by the methods and laws given heretofore for the treatment of simple alternating or sine-wave problems.* Problems numbered 47 and 48 illustrate this method of solving problems that arise in dealing with alternating quantities.

Effective Value of a Non-sine Alternating Curve.—The effective value of an alternating curve is the square root of the mean square of its values. The equation of the general alternating curve is

$$\begin{aligned} y = {} & A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \ldots + \\ & B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \ldots \end{aligned} \tag{19}$$

Squaring,

$$\begin{aligned} y^2 = {} & A_1^2 \sin^2 x + A_2^2 \sin^2 2x + A_3^2 \sin^2 3x + \ldots + \\ & B_1^2 \cos^2 x + B_2^2 \cos^2 2x + B_3^2 \cos^2 3x + \ldots \end{aligned} \tag{20}$$

+ terms of the form

$$\begin{aligned} & (A_m \sin mx)(A_n \sin nx) + \ldots + \\ & (A_n \sin nx)(B_n \cos nx) + \ldots + \\ & (B_m \cos mx)(B_n \cos nx) + \ldots \end{aligned}$$

* From this it is seen that methods employed for the treatment of sine waves are necessary for the convenient solution of all non-sine waves. Such methods have, therefore, an entirely general application.

All of these product terms have a mean value of zero. See Section 13, page 45, law 1.

From Section 10, *f*, 1.

$$(\text{Mean } A^2 \sin^2 x) = \tfrac{1}{2}A^2. \quad . \quad . \quad . \quad . \quad (21)$$

From which it follows that

(Mean y^2)

$$= \tfrac{1}{2}(A_1^2 + A_2^2 + A_3^2 + \ldots + B_1^2 + B_2^2 + B_3^2 + \ldots). \quad (22)$$

The effective value is, therefore,

Effective y

$$= \frac{1}{\sqrt{2}}\sqrt{A_1^2 + A_2^2 + A_3^2 + \ldots B_1^2 + B_2^2 + B_3^2 + \ldots}). \quad (23)$$

Since $\tfrac{1}{2}A^2$ and $\tfrac{1}{2}B^2$ are the mean squares of their corresponding curves, the result may be further reduced. The mean square of any harmonic is

$$H_n^2 = \tfrac{1}{2}(A_n^2 + B_n^2), \text{ (see Sec. 11, } c,) \quad . \quad . \quad (24)$$

where H_n is the effective value of the harmonic.

Substituting,

$$\text{Effective } y = \sqrt{H_1^2 + H_2^2 + H_3^2 + \ldots} \quad . \quad . \quad (25)$$

The effective value of a general periodic alternating curve is the square root of the sum of the squares of the effective values of its harmonic components.

Terminology of Alternating Quantities.—In the general equation of the sine curve,

$$y = a \sin (x + \alpha),$$

if the angle x increases at a uniform rate, the curve is said to be **periodic.** As the curve represents an alternating quantity, the expression **alternation** is used to designate a half wave which is generated while the ordinate varies in value from

maximum in one direction to maximum in the other direction. Two consecutive alternations constitute a complete **cycle,** and the time occupied in passing through a complete cycle is a **period.** This is the time during which x increases by 360°. The number of *cycles per second* or the number of *periods per second* is called the **periodicity,** or more commonly the **frequency.** In the treatment of alternating currents the term **wave** is generally used in lieu of the expression periodic alternating curve or quantity.

CHAPTER IV.

COMPLEX QUANTITIES.

SYNOPSIS.

14. **Vectors.**—*a. Definitions.*—A **vector** or **directed quantity** is a quantity having both magnitude and direction. A vector is represented graphically by a line which has a length representing the magnitude of the quantity, and which lies in the direction of the vector. The line is usually drawn with an arrow-head to indicate its direction.

A vector can only be represented analytically by an expression which takes account of its direction as well as its magnitude. Such an expression is a *complex quantity.*

b. Graphical Conventions.—The position of a vector is specified by reference to a horizontal line which passes through its initial end. If the vector coincides with this horizontal line, and extends from left to right, it is in the zero position, and may be represented by a positive numeral. If the vector is horizontal, but extends from right to left, it is negative, and may be represented by a negative numeral. If a vector be multiplied by the factor (-1), it is reversed in direction.

The process of reversal is assumed to consist in a counter-clockwise rotation of 180° about the initial end of the vector as a centre. If a vector be multiplied twice by $\sqrt{-1}$, i.e., by $(\sqrt{-1})^2$, or by (-1), it is rotated 180°. It is logical to assume that each factor produces half of this rotation, or that multiplying a vector by $\sqrt{-1}$ gives to it a positive rotation of 90°. For convenience in algebraic expression, the letter j is used as the equivalent of $\sqrt{-1}$. Then

$$j = \sqrt{-1},$$
$$-j = -\sqrt{-1},$$
$$j^2 = -1.$$

The operation of division is the reverse of that of multiplication. Thus the division of a vector by j is equivalent to giving it a negative rotation of 90°, and is represented by the factor $-j$. Thus,

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j,$$
$$j = \frac{1}{-j}.$$

The quantity $a + jb$, Fig. 46, is a vector which is the sum of a positive horizontal line, a, and a positive vertical line, jb. The quantity $a - jb$ is a vector which is the sum of a positive horizontal line, a, and a negative vertical line, $-jb$.

Summary.—1. A vector which is horizontal and positive is denoted by a quantity giving its length only. If horizontal and negative, the quantity is preceded by the minus sign.

2. A vector which is vertical and positive is denoted by a quantity giving its length, and by the factor j. If vertical and negative, this product is preceded by the minus sign.

3. A vector which is neither horizontal nor vertical is expressed as the sum of its horizontal and vertical components.

A quantity not containing the factor j is *real.*

A quantity containing the factor j is *imaginary.*

A quantity made up of real and imaginary components is a **complex quantity.**

c. Addition of Complex Quantities.—Vectors may be added conveniently by the addition of the corresponding complex

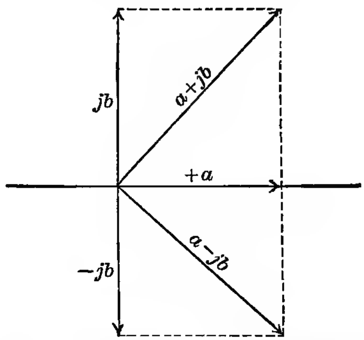


FIG. 46.—Diagram of Complex Quantities.

quantities, from which the absolute value as well as the phase position is readily obtained. For example, the complex quantity representing the sum of the vectors $(a+jb)$ and $(c+jd)$ is

$$(a+jb)+(c+jd)=(a+c)+j(b+d). \quad . \quad (26)$$

The absolute length of the vector is

$$l=\sqrt{(a+c)^2+(b+d)^2}. \quad . \quad . \quad . \quad . \quad (27)$$

This occupies a phase position with respect to the X axis which is found as follows:

$$\alpha=\sin^{-1}\frac{b+d}{\sqrt{(a+c)^2+(b+d)^2}}. \quad . \quad . \quad (28)$$

Numerical Illustrations.—Prob. 13. Draw the vectors $5, -5, j5, 3+j4, 2-j6, 3+j7, 5+j6$.

Prob. 14. Add the following vectors, and draw the vector representing their sum; also draw the vectors separately and add them graphically.

(a)....	$(2+j8)$	$(4-j3)$	$(1+j6)$.
(b)....	$(3-j5)$	$(9+j6)$	$(3-j2)$.
(c)....	$(-4+j1)$	$(-6-j8)$	$(-9+j7)$.

d. Multiplication and Division of Complex Quantities.—Complex quantities are multipled and divided like ordinary binomials. It is only necessary to bear in mind the meaning of the factor j. Whenever j^2 occurs, the minus sign may be substituted. Thus,

$$\begin{aligned}(a+jb)^2 &= a^2+j2ab+j^2b^2\\ &= a^2+j2ab-b^2,\\ (a+jb)(r+jx) &= ar+jbr+jax+j^2bx\\ &= ar+j(br+ax)-bx. \qquad (29)\end{aligned}$$

This product may be written

$$(ar-bx)+j(br+ax). \quad . \quad . \quad . \quad . \quad (30)$$

Its absolute value is, therefore,

$$\sqrt{(ar-bx)^2+(br+ax)^2} \quad . \quad . \quad . \quad . \quad (31)$$

After expanding we may reduce this value to the form

$$\sqrt{a^2+b^2}\times\sqrt{r^2+x^2}. \quad . \quad . \quad . \quad . \quad (32)$$

From this result it follows that the product of the absolute values of two vectors is equal to the absolute value of their product.

Let θ', θ'', and θ, respectively, be the angles which

$$a+jb,$$
$$r+jx,$$

and their product,

$$(ar-bx)+j(br+ax),$$

make with the horizontal. Then

$$\cos\theta' = \frac{a}{(a^2+b^2)^{\frac{1}{2}}},$$

$$\cos\theta'' = \frac{r}{(r^2+x^2)^{\frac{1}{2}}},$$

$$\sin\theta' = \frac{b}{(a^2+b^2)^{\frac{1}{2}}},$$

$$\sin\theta'' = \frac{x}{(r^2+x^2)^{\frac{1}{2}}},$$

$$\cos\theta = \frac{ar - bx}{(a^2+b^2)^{\frac{1}{2}} \times (r^2+x^2)^{\frac{1}{2}}},$$

$$\cos(\theta' + \theta'') = \cos\theta' \cos\theta'' - \sin\theta' \sin\theta''.$$

Substituting, we have, after reducing,

$$\cos(\theta' + \theta'') = \frac{ar - bx}{(a^2+b^2)^{\frac{1}{2}} \times (r^2+x^2)^{\frac{1}{2}}}, \quad . \quad . \quad (33)$$

wherefore

$$\theta = \theta' + \theta''.$$

From this it follows that the phase angle of the product of two vectors is equal to the sum of their individual phase angles. *Phase angle* is the angle which the vector makes with the line of reference, usually taken as the horizontal.

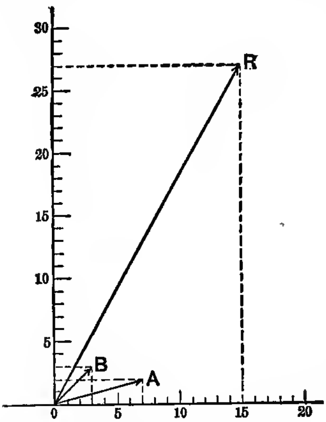


FIG. 47.—The Multiplication of Vectors.

The quantity resulting from the multiplication or division of complex quantities may be laid down graphically. Thus, in Fig. 47, the vectors OA and OB are to be multiplied. Expressed analytically, these vectors are $7 + j2$ and $3 + j3$. Their product is $21 + j6 + j21 - 6 = 15 + j27$.

The numerical or absolute values of OA, OB, and OR are

$$\sqrt{7^2 + 2^2} = 7.27, \quad \sqrt{3^2 + 3^2} = 4.24,$$

and

$$\sqrt{15^2 + 27^2} = 30.8 = 7.27 \times 4.24.$$

The absolute value of the product of two or more vectors is the product of the absolute values of the vectors. The angle which this product makes with the initial line is the sum of the angles made by the several vectors.

As the operation of division is the reverse of that of multiplication, a similar law may be stated for the division of complex quantities.

The absolute value of the quotient, when one vector is divided by another, is obtained by dividing the absolute value of the dividend by that of the divisor. The angle which this quotient makes with the initial line is the difference of the angles made by the two vectors with the initial line.

As a corollary it may be stated that the reciprocal of a vector is the reciprocal of its absolute value with its angle reversed. Thus, in Fig. 48, *OA′* is the reciprocal of *OA*.

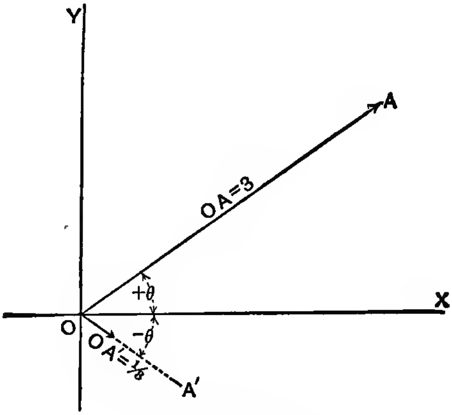


FIG. 48.—Division of Vectors.

15. Alternating Quantities and Vectors.—*a. Alternating Quantities Represented by Vectors.*—The actual value of an

alternating quantity is changing from moment to moment, and if the positive and negative alternations are equal the average value for any whole number of alternations is zero. The average *effect* produced by such a quantity is not, however, necessarily zero. An instance is found where an alternating electric current is present in a conductor. The average current is zero, but the effect of the current is to heat the conductor continuously. A new definition of **effective value** may now be given. It will be shown later that this is the same quantity which was defined on page 29 when the alternating quantity is electric current or e.m.f.

The effective value of an alternating quantity is its value measured in terms of an average effect which it may produce continuously.

Such a quantity, being an average value, may be expressed by a positive numeral or represented by a straight line. It is not in the strictest sense a directed quantity or vector. It is nevertheless a very great convenience in many cases to arbitrarily assign to an effective value a direction which shall indicate the phase position of the alternating quantity itself. The alternating quantity is then represented graphically by a vector having a length equal to the effective value of the quantity, and a direction representing its phase position. The phase position is a relative item, and is specified in terms of the phase position of some other alternating quantity of the same frequency, which is chosen as a standard of reference. This standard is always represented by a vector in the zero position.

b. Operations on Vectors Representing Alternating Quantities.—Alternating quantities may be added or subtracted by adding or subtracting their respective vectors. The result of such an operation is represented by the resulting vector.

Independent alternating quantities may *not* be multiplied

or divided by performing these operations on vectors representing their effective values. Such an operation is meaningless and the vector resulting from it does *not* represent the result of the multiplication or division of the alternating quantities.

A vector representing an effective value may be multiplied or divided by a vector representing a constant quantity having magnitude and direction, i.e., by a simple vector. The result of such an operation is a vector which represents the result of a similar operation on the quantities themselves.

It is often very convenient to determine the result of operations performed on alternating quantities by performing the same operations on vectors representing their effective values. This method is used for convenience only, and its limitations, as given above, must always be kept in mind. Illustrations of the statements of this section will be given in the course of the discussions of alternating currents.

CHAPTER V.

LAWS OF THE ELECTRIC CIRCUIT.

SYNOPSIS.

16. Consumption of Electromotive Force in Single Circuits.—*a. Impressed E.m.f. Consumed in Resistance,*

$$E = Ir. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

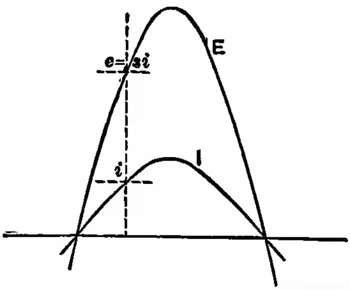


FIG. 49.—Showing E.m.f. Consumed in Resistance.

Resistance consumes e.m.f. in direct proportion to the value of the current. It follows that where the current has the sine form, the e.m.f. consumed, which is proportional to the current at all instants, must also give a sine wave when plotted. Thus, in Fig. 49, let the sine wave of current, I, be established through a resistance of three ohms. The e.m.f. at any instant is $ir = 3i$, which when plotted in the same manner gives a sine wave, E, having three times the height of the

current wave. Under these circumstances the e.m.f. and current are in phase.

When an alternating current is established through a circuit containing resistance only, it is in phase with the impressed pressure which establishes it.

A current which is alternating has the value of unity when it liberates heat at the same rate as would a unit direct current if passed through the same resistance. Since the rate at which heat is liberated at any instant is proportional to the square of the instantaneous value of the current, it follows that the average square of the instantaneous values of an alternating current is equal to the square of a continuous current which has the same heating value. Therefore the "square root of the mean square," or the *effective value* of an alternating current (see Section 10, *f*), is the value commonly used. For an alternating current in a circuit containing resistance only, Ohm's law holds for effective values of current and e.m.f., as well as for instantaneous values. The effective e.m.f. consumed by resistance is

$$E = Ir.$$

Prob. 15. An electric heater has a resistance of 18 ohms. How much e.m.f. will be consumed in it when it carries a current of 6 amperes?

Solution: $E = Ir = 6 \times 18 = 108$ volts. *Ans.*

b. Impressed E.m.f. Consumed in Inductance.

$$E = j2\pi fLI.$$

From the definition for the unit of inductance, the e.m.f. due to an inductance of L units is L times the rate of change of current in the circuit, or

$$e = -L\frac{di}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

In this instance e and $\frac{di}{dt}$ have opposite signs, since the e.m.f. of inductance always opposes a change in the current.

Assuming i to be a sine wave of current,

$$i = A \sin \frac{2\pi}{T}t, \quad \ldots \ldots \quad (36)$$

whence

$$\frac{di}{dt} = A\frac{2\pi}{T} \cos \frac{2\pi}{T}t \quad \ldots \ldots \quad (37)$$

and

$$e = -LA\frac{2\pi}{T} \cos \frac{2\pi}{T}t. \quad \ldots \ldots \quad (38)$$

That is, e is a sine wave of pressure 90° behind the current wave. An equal and opposite wave of e.m.f. is *consumed* by the inductance.

Letting e represent the consumed e.m.f., it may be written

$$e = LA\frac{2\pi}{T} \cos \frac{2\pi}{T}t, \quad \ldots \ldots \quad (39)$$

which is a pressure wave 180° ahead of the self-induced e.m.f., or 90° ahead of the current.

The component of impressed e.m.f. which is consumed by the inductance of a circuit is 90° ahead of the current flowing in the circuit.

The maximum value of this e.m.f. is $e_{\max.} = LA\frac{2\pi}{T}$.

But the maximum value of the current is $i_{\max.} = A$. Therefore

$$e_{\max.} = Li_{\max.} \frac{2\pi}{T}.$$

Dividing this equation by $\sqrt{2}$, and writing E for $\frac{e_{\text{max.}}}{\sqrt{2}}$ and I for $\frac{i_{\text{max.}}}{\sqrt{2}}$, this becomes

$$E = \frac{2\pi}{T} LI,$$

where E and I are effective values of e.m.f. and current respectively.

Or, writing $f = \frac{1}{T}$,

$$E = 2\pi f LI, \quad \ldots \ldots \quad (40)$$

where f is the frequency.

As the vector representing the pressure wave is 90° in advance of that representing the current wave, this is indicated by the factor j, thus:

$$E = j2\pi f LI.$$

The relations given in this equation are illustrated in Fig. 50.

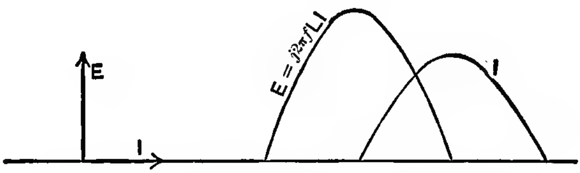


FIG. 50.—Showing E.m.f. Consumed in Inductance.

E.m.f. developed by unit current in an electric circuit in quadrature with it is called **reactance.**

Thus, in a circuit containing inductance, the reactance is equal to the volts per ampere developed or consumed by the inductance. Its value in such a circuit is $2\pi f L$.

Prob. 16. What will be the consumption of e.m.f. in a circuit of .2 henry inductance and in which the current has an effective value of 5 amperes and a frequency of 60 periods per second?

Solution: Substituting in the expression $E = j2\pi fLI$,

$$E = j2\pi \times 60 \times .2 \times 5 = j377 \text{ volts},$$

where j indicates that the e.m.f. of 377 volts is 90° in advance of the current.

c. Impressed E.m.f. Consumed in Capacity

$$E = -j\frac{1}{2\pi fC}\cdot I.$$

When the electric circuit is closed through a condenser whose capacity is C farads the developed e.m.f. is

$$e = -\frac{Q}{C},$$

where Q is the charge in coulombs present in the condenser.

$$\frac{de}{dt} = -\frac{\frac{dQ}{dt}}{C}. \quad \ldots \ldots \quad (41)$$

Since $\frac{dQ}{dt} = i$, and $i = i_{\max.} \sin \frac{2\pi}{T}t$,

by substitution

$$\frac{de}{dt} = -\frac{1}{C}i_{\max.} \sin \frac{2\pi}{T}t. \quad \ldots \ldots \quad (42)$$

From this it is seen that the current divided by the capacity equals the rate of change of the e.m.f. which the current pro-

duces due to the reactance of the circuit. The maximum value of this e.m.f. will, therefore, be 90° in advance of the current and its value will be

p 42

$$\frac{de}{dt} = \frac{2\pi}{T} e_{max.} \sin\left(\frac{\pi}{2} + \frac{2\pi}{T}\cdot t\right). \quad \text{(See Sec. 11, } e\text{, eq. (13)).} \quad (43)$$

Substituting,

$$\frac{2\pi}{T} e_{max.} \sin\left(\frac{\pi}{2} + \frac{2\pi}{T}\cdot t\right) = -\frac{i_{max.}}{C} \sin\frac{2\pi}{T}\cdot t.$$

The corresponding effective values are

$$j\frac{2\pi}{T}E = -\frac{1}{C}I.$$

Let $\frac{1}{T}$ be f. Then

$\frac{1}{j} = -j$

$$E = j\frac{1}{2\pi f C}I. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The impressed e.m.f. *consumed* by the capacity reactance is opposite to the reactance e.m.f., or is

$$E = -j\frac{1}{2\pi f C}I.$$

The component of impressed e.m.f. which is consumed by the capacity reactance of a circuit is 90° behind the current flowing in the circuit.

These relations are shown in Fig. 51.

Prob. 17. What will be the consumption of e.m.f. in a circuit which has a capacity of 2 microfarads, when a current of .5 ampere with a frequency of 60 p.p.s. is passed through it?

Solution: Substituting in the expression

$$E = -j\frac{1}{2\pi fC}I,$$

$$E = -j\frac{1}{2\pi \cdot 60 \cdot .000002} \cdot 5$$

$$= -j663 \text{ volts.} \quad \textit{Ans.}$$

The factor $-j$ indicates that the e.m.f. of 663 volts is 90° behind the current.

d. Impressed E.m.f. Consumed by Resistance and the Reactances of Inductance and Capacity in Series.—By combining the results of Sections *a*, *b*, and *c*, the e.m.f. consumed in a circuit

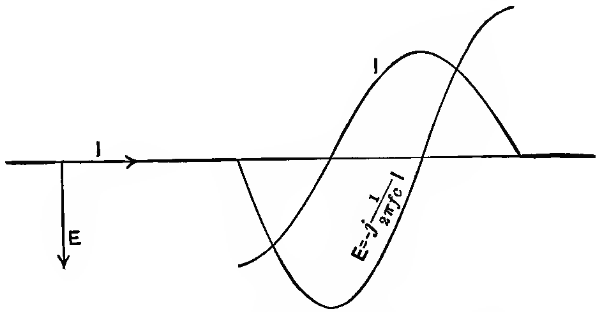


FIG. 51.—Showing E.m.f. Consumed in Capacity.

containing any combination of resistance, inductance, and capacity in series may be obtained. Thus, for the general case,

$$E = rI + j2\pi fLI - j\frac{1}{2\pi fC}I. \quad . \quad . \quad . \quad (45)$$

To simplify this expression the letter x' is substituted for $2\pi fL$, and x'' is substituted for $\frac{1}{2\pi fC}$. Then

$$E = I(r + jx' - jx''). \quad . \quad . \quad . \quad . \quad (46)$$

These relations are shown in Fig. 52.

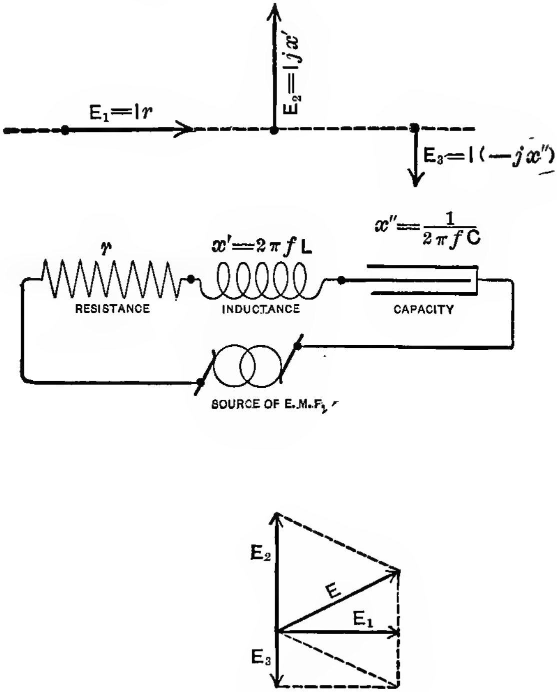


FIG. 52.

If I is the effective current in amperes, it is seen from the equation that the quantity in brackets is the "*volts per ampere*" consumed in the circuit. This quantity is termed the **impedance** of the circuit.

If the impedance of a circuit be denoted by z, this equation may be written

$$E = Iz, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

or

$$I = \frac{E}{z}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

which is analogous to Ohm's law.

17. Problems in Single Series Circuits.—It will be noted that in single series circuits the current is the same throughout and the e.m.f.s are added. The various e.m.f.s may be conveniently located in phase position by reference to the current. All of the quantities in this equation may be treated as vectors.

Graphical solutions of the following problems will ordinarily be most instructive, and if the construction be carefully made, the results obtained by the graphical method will be sufficiently accurate. When the algebraic solution is attempted, the student should carry along an *approximate* graphical solution at the same time by means of a free-hand sketch. In this manner errors in determining the phase relation of current and e.m.f. may be largely eliminated.

All problems in simple electric circuits, where either current, e.m.f., or impedance is to be determined, may be solved by means of the general equation

$$I = \frac{E}{z},$$

or, for the particular case of a continuous current, by the equation

$$I = \frac{E}{r}.$$

In the general case, the effective values of I and E are dealt with, and these may be represented by vectors. Since the impedance, z, is a vector which relates I and E, it follows that any of the operations necessary to the solution of equation (45) may be performed upon the vectors representing the quantities, and the result will be a vector representing the true result of the operation. (See Section 15.)

This fact makes possible the ready solution of problems which would otherwise present considerable mathematical difficulties. The solution may either be graphical, in which

case the vectors are drawn to scale, or analytical, in which case the results of the geometrical operations are obtained by making use of the algebra of complex quantities. Both methods are illustrated in the solutions that follow.

Prob. 18. When the e.m.f. applied at the terminals of an incandescent lamp is 110 volts and the resistance of the lamp is 200 ohms, what current will flow?

Solution:

$$I = \frac{E}{z}; \quad z = r = 200 \text{ ohms.}$$

$$I = \frac{110}{200} = .55 \text{ ampere.} \quad \textit{Ans.}$$

Prob. 19. A current of 105 amperes is furnished by the armature of a railway generator that has a resistance of .15 ohm. What e.m.f. is consumed in the armature?

Solution:

$$E = zI = .15 \times 105 = 15.75 \text{ volts.} \quad \textit{Ans.}$$

Prob. 20. What e.m.f. must be furnished at a frequency of 60 cycles per second to set up a current of 8 amperes through a resistance of 175 ohms and an inductance of .5 henry?

Solution:

$$z = r + jx'.$$

$$\begin{aligned} E = Iz = I\cdot(r + jx') &= 8\cdot(175 + j2\pi\cdot 60.5) \\ &= 1400 + j1508 \\ = I\sqrt{r^2 + x'^2} &= 2059, \ \underline{/\tan^{-1}\frac{1508}{1400}}, \text{ volts} \\ &= 2059, \ \underline{/47^\circ\ 8'}, \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

The symbol $\underline{/\quad}$ indicates that the angle of 47° 8′ is positive, or that the e.m.f., 2059 volts, *leads* the current by

an angle of 47° 8′. The same symbol inverted, $\overline{\diagdown}$, is used to denote a negative angle.

The significance of the results obtained above is shown graphically in Fig. 53. The sum of the e.m.f.s OE_r consumed in resistance and $OE_{x'}$ consumed in inductance gives the e.m.f. OE, occupying a time position 47° 8′ ahead of the e.m.f. OE_r, which is in phase with the current. The alternating pressure applied to the circuit is numerically equal to 2059 volts and leads the current which it establishes by 47° 8′.

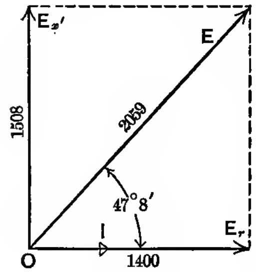


FIG. 53.

The phase position of the current in this diagram is given by the vector having the closed arrow-head. It is drawn to a larger scale than the e.m.f. values.

Prob. 21. At what pressure and at what angle of lag will the current in the preceding problem be established when the frequency is (*a*) 125 cycles? (*b*) 25 cycles?

(*a*) 3442, $\underline{/65° \; 58'}$, volts. *Ans.*

(*b*) 1534, $\underline{/24° \; 7'}$, volts. *Ans.*

Prob. 22. An alternator develops a terminal pressure of 2000 volts at a frequency of 60 cycles. What current will flow if the circuit contains a resistance of 100 ohms and a capacity of 50 microfarads?

Solution:

$$I = \frac{E}{z} = \frac{E}{r - jx''} = \frac{2000}{100 - j\dfrac{1}{2\pi \cdot 60 \cdot 50 \cdot 10^{-6}}}$$

$$= \frac{2000}{100 - j53} = \frac{2000}{112.7 \overline{\diagdown 28°}} = 17.75, \underline{/28°}, \text{ amperes. } \textit{Ans.}$$

The elements of this problem are given graphically in the diagram Fig. 54. The alternating current generator establishes through the resistance of 100 ohms and the condenser having a capacity of 50 microfarads, or 53 ohms of reactance at 60 cycles, a current of 17.75 amperes in a phase position of 28° in advance of the generator pressure. The resistance consumes 1775 volts in phase and the condenser 940 volts in lagging quadrature with the current. The quadrature sum of these two pressures equals the generator pressure of 2000 volts.

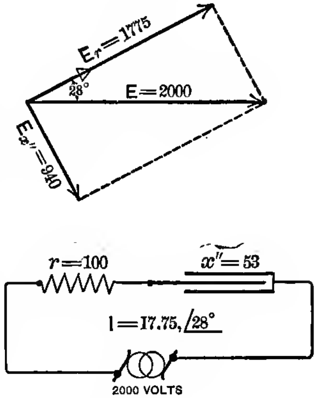


FIG. 54.

Prob. 23. What are the corresponding solutions of the preceding problem when the dynamo frequency is (*a*) 125 cycles? (*b*) 25 cycles?

(*a*) 19.4, $\angle 14.25°$, amperes. *Ans.*

(*b*) 12.3, $\angle 51.85°$, amperes. *Ans.*

Prob. 24. In Fig. 55 the dynamo, A, generates a total pressure of 2200 volts at a frequency of 125 cycles. The resistance of its armature is 1 ohm, and its inductance is .01 henry. $L = .05$ henry, $R = 50$ ohms, and $C = 20$ microfarads. What is the value of the current?

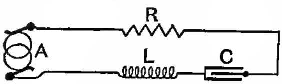


FIG. 55.

Solution: Total impedance =

$$(1 + 50) + j(.05 + .01)2\cdot\pi\cdot 125 - j\left(\frac{1000000}{20\cdot 2\pi\cdot 125}\right)$$
$$= 51 + j47.124 - j63.661$$
$$= 51 - j16.537$$

$$= 53.614, \overline{\diagdown \tan^{-1} .3243}$$

$$= 53.614, \overline{\diagdown 17^\circ 58'}, \text{ ohms;}$$

$$\text{current} = \frac{2200}{53.614 \overline{\diagdown 17^\circ 58'}} = 41.034, \underline{/ 17^\circ 58'}, \text{ amperes. } Ans.$$

18. Problems in Simple Multiple Circuits.—In simple multiple circuits the various branches of the combination are subjected to the same e.m.f., but the separate currents and their phase positions with respect to the e.m.f. depend upon the impedances of the individual branch circuits. Hence the various currents are referred to the e.m.f. as a phase standard, for the e.m.f. is the same for all the circuits. The total current is the geometrical sum of the separate currents.

In determining the total impedance of a combination of this kind the *reciprocals* of the impedances of the branch circuits are added, with due regard to their relative phase positions. These reciprocals of the impedance are known as the **conductivities** of the circuits, hence the measure of the conductivity of a circuit is *the number of amperes set up in it by one volt of applied pressure.* This is in contrast to the **impedance** of the circuit, the measure of which is *the number of volts required to set up one ampere.* When the combined conductivity of several branch circuits has been obtained, the corresponding total impedance is simply its reciprocal. This is illustrated in the following problems.

Prob. 25. Two incandescent lamps, having resistances of 100 ohms and 200 ohms, are in multiple across 110-volt mains. What is the impedance of the combination, and what is the strength of the total current?

Solution: The conductivity of branch (1) is $\frac{1}{100}$, that of branch (2) $\frac{1}{200}$, and the total conductivity is $\frac{3}{200}$. The joint

resistance is, therefore, $\frac{200}{3}$, or 66.66 ohms, and the total current is $\frac{110}{66.66} = 1.65$ amperes. *Ans.*

Prob. 26. A pressure of 1000 volts at 60 cycles is applied at each of the terminals of R, L, and C, Fig. 56. $R = 120$ ohms, $L = .5$ henry, and $C = 25$ microfarads. The joint impedance and total strength of established current are required. Two solutions will be given.

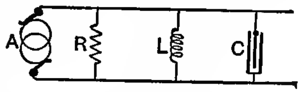


FIG. 56.

First Solution:

$$I_1 = \frac{1000}{120} = 8.33 \text{ amperes,}$$

$$I_2 = \frac{1000}{j \cdot 2\pi \cdot 60 \cdot .5} = -j5.305 \text{ amperes,}$$

$$I_3 = \frac{1000}{-j\frac{1}{2\pi \cdot 60 \cdot 25 \cdot 10^{-6}}} = j9.425 \text{ amperes.}$$

$$\Sigma I_1,\ I_2,\ I_3 = 8.33 + j4.12 = 9.30, \underline{/26^\circ 19'}, \text{ amperes.} \quad \textit{Ans.}$$

$$z = \frac{1000}{9.30 \ \underline{/26^\circ 19'}} = 107.37, \overline{\diagdown 26^\circ 19'}, \text{ ohms.} \quad \textit{Ans.}$$

Second Solution: The impedances of the several branches are, respectively,

$$120 \text{ ohms,}$$

$$j \cdot 2\pi \cdot 60.5 = j189 \text{ ohms,}$$

and

$$-j\frac{1}{2\pi \cdot 60 \cdot 25 \cdot 10^{-6}} = -j105 \text{ ohms.}$$

The conductivities are the reciprocals of these values, or

$$.0083 \text{ mhos,}$$

$$-j\,00529 \text{ mhos,*}$$

* For definition of *mho*, see page 83.

and

$$j.00943 \text{ mhos, respectively.}$$

The total conductivity is the sum of the separate conductivities, or is

$$00833 - j00529 + j00943, \quad = .009, \underline{/26^\circ\ 19'}, \text{ mhos,}$$

or amperes per volt.

The impedance of the branched portion of the circuit is, therefore,

$$z = \frac{1}{.0093\underline{/26^\circ\ 19'}} = 107.5, \overline{\diagdown 26^\circ\ 19'}, \text{ volts per ampere.}$$

The current is

$$I = \frac{E}{z} = 9.30, \underline{/26^\circ\ 19'}, \text{ amperes.} \quad \textit{Ans.}$$

Prob. 27. The dynamo armature in the preceding problem has a resistance of 5 ohms and an inductance of .05 henry. What is the total impedance of the circuit and what e.m.f. is developed by the dynamo?

The impedance of the armature is $5 + j18.9$ ohms, and the total impedance is $5 + 96.2 + j(18.9 - 47.9) = 101.2 - j29$

$$= 104.3, \overline{\diagdown 15^\circ\ 59'}, \text{ ohms.} \quad \textit{Ans.}$$

The e.m.f. developed by the dynamo is 104.3×9.3

$$= 969, \overline{\diagdown 15^\circ\ 59'}, \text{ volts.} \quad \textit{Ans.}$$

19. Consumption of Electromotive Force in Single and Multiple Circuits in Series.—From the solution of the foregoing problems it is seen that:

a. Single circuits are treated by means of their *impedances.*

b. Multiple circuits are treated by means of their *conductivities;* i.e., the reciprocals of the individual impedances that are in multiple.

c. Single and multiple circuits in series are treated as in (*a*) and (*b*). *The impedance of a single circuit* which is the *equivalent of the multiple circuit* is then determined. The final solution is, therefore, that of an equivalent single circuit by means of impedances in series.

For practical convenience the term **ohms** is now used to denote the **volts per ampere** consumed in a circuit **by impedance,** although the *unit ohm* is defined as the resistance which consumes one volt per ampere.

Likewise the term **mhos** is used to denote the **amperes per volt** established in an electric circuit, although the *unit mho* is defined as the unit of conductivity equal to the *reciprocal* of the *ohm.*

Prob. 28. In the circuit of Fig. 57 the constants are as follows: $L = .1$ henry, $R = 20$ ohms, $R' = 30$ ohms, $C = 50$ microfarads. The resistance of the line is negligible. Find the total impedance of the circuit external to the dynamo at a frequency of 50 cycles per second.

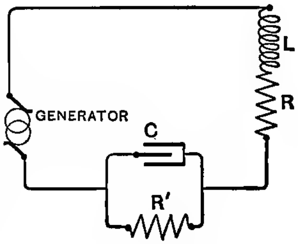


FIG. 57.

Solution:

$$\text{Impedance of } L = \quad j2\pi fL \quad = j31.416 \text{ ohms.}$$
$$\text{" \quad " } R = \quad 20 \quad \text{"}$$
$$\text{" \quad " } R' = \quad 30 \quad \text{"}$$
$$\text{" \quad " } C = \frac{1}{j2\pi fC} = -j63.69 \quad \text{"}$$

Since the two branches R' and C'' are in multiple, their joint impedance must be determined. This is the reciprocal of the joint conductivity.

$$\text{Conductivity of } R' = .0333 \text{ mhos.}$$
$$\text{" \quad " } C = j.0157 \quad \text{"}$$

Joint conductivity of R' and $C =$

$$.0333 + j.0157, \quad = .0368, \underline{/ \tan^{-1} \frac{.0157}{.0333}}, \text{ mhos}$$

$$= .0368, \underline{/ 25^\circ 15'}.$$

Joint impedance of R' and $C =$

$$\frac{1}{.0368 \underline{/ 25^\circ 15'}} = 27.17, \overline{\diagdown 25^\circ 15'}, \text{ ohms.}$$

In order to add this quantity to the impedances of L and R it is best reduced to components.

$$27.17 \overline{\diagdown 25^\circ 15'} = 24.56 - j11.58.$$

The total impedance is the sum of all the impedances in series, or is

$$z_b = j31.416 + 20 + 24.56 - j11.58$$

$$= 44.56 + j19.836 = 48.8, \underline{/ \tan^{-1} \frac{19.836}{44.56}}, \text{ ohms.}$$

$$= 48.8, \underline{/ 24^\circ 1'}, \text{ ohms.} \quad \textit{Ans.}$$

Prob. 29. If the dynamo in Fig. 57 delivers a terminal pressure of 2000 volts, what current will flow in the circuit, and what is its phase position relative to the terminal pressure?

Solution: The current in amperes is equal to the impressed pressure divided by the impedance. Taking the phase of the impressed pressure as the phase of reference,

$$I = \frac{2000}{48.8 \tan^{-1}\left(\frac{19.8}{44.6}\right)} = 40.9 \tan^{-1}\left(-\frac{19.8}{44.6}\right), \text{ amperes,}$$

$$= 40.9, \overline{\diagdown 24^\circ 1'}, \text{ amperes.} \quad \textit{Ans.}$$

That is, the current lags behind the impressed pressure by an angle of $24^\circ 1'$.

Prob. 30. Find the e.m.f. at the ends of L, Fig. 57, and find its phase position both with respect to the current flowing and with respect to the e.m.f. at the dynamo terminals.

Solution: The current flowing is 40.9 amperes. Its phase position with respect to the dynamo pressure is lagging by the angle

$$\tan^{-1}\frac{19.8}{44.6} = 24^\circ\ 1'.$$

The pressure used up in sending this current through l is

$$E_l = 40.9 \times j31.416 = j1284.9 \text{ volts.}$$

The factor j indicates that this pressure is 90° ahead of the current flowing. Since the current is 24° 1′ behind the dynamo pressure, it follows that the pressure E_l, of 1284.9 volts, is

$$(90^\circ - 24^\circ\ 1') = 65^\circ\ 59'$$

ahead of the dynamo pressure. *Ans.*

Prob. 31. Find the e.m.f. with its phase position relative to the current, (*a*) at terminals of r; (*b*) for multiple portion of circuit.

Solution: (*a*)

$E_r = 20 \times 40.9 = 818$ volts, in phase with the current. *Ans.*

(*b*)

$$E_m = 40.9 \times 27.17;\ \tan^{-1}\left(-\frac{.0157}{.0333}\right)$$

$$= 1111.25,\ \overline{\diagdown 25^\circ\ 15'}, \text{ volts.} \quad \textit{Ans.}$$

Check: Since the total pressure at the dynamo terminals is used up in l, r, and the multiple branch, the sum of the separate e.m.f.s found for these portions should give 2000 volts, leading the current 24° 1′.

The sum is

$$j1284.9 + 818 + 1004.5 - j473.6 = 1822.5 + j811.3,$$

or 2000;

$$\tan^{-1}\frac{811.3}{1822.5} = 2000,\ \underline{/24^\circ\ 1'}, \text{ volts.} \quad \textit{Ans.}$$

Prob. 32. If the generator armature, Fig. 57, has resistance of 4 ohms and an inductance of .008 henry, what is its impedance?

Solution:

$$z_a = 4 + j2\pi \cdot 50 \cdot .008$$
$$= 4 + j2.51 \text{ ohms},$$

or $$4.72, \underline{/32^\circ\ 7'}, \text{ ohms.} \quad \textit{Ans.}$$

Prob. 33. Using the answers to problems 28 to 32, find the impedance of the complete closed circuit, including the generator, and find the *total* dynamo e.m.f.

Solution: The *total* impedance is

$$z_b = 44.56 + j19.84 + 4 + j2.51$$
$$= 48.56 + j22.35, \quad \text{or} \quad 53.45, \underline{/24^\circ\ 40'}, \text{ ohms.} \quad \textit{Ans.}$$

The total dynamo pressure is

$$40.9 \times 53.45\ \underline{/24^\circ\ 40'} = 2186, \underline{/24^\circ\ 40'}, \text{ volts.} \quad \textit{Ans.}$$

Prob. 34. An alternator is delivering 250 K.W. of electrical power to incandescent lamps over a transmission line at 12,000 volts pressure, 50 p.p.s., and the current lags 25° 50′ behind the e.m.f. Neglecting the impedance of the generator, find (*a*) the current and (*b*) the impedance of the circuit.

(*a*) $23.15, \overline{\diagdown 25^\circ\ 50'}$, amperes.
(*b*) $518.4, \underline{/25^\circ\ 50'}$, ohms. *Ans.*

Prob. 35. Find (*a*) the resistance; (*b*) the reactance; (*c*) the inductance of the circuit in the preceding problem, assuming the capacity of the line to be zero.

(*a*) 466.56 ohms. *Ans.*
(*b*) $j225.89$ ohms. "
(*c*) .719 henry. "

Prob. 36. The "charging current" * on one of the underground cables of Cornell University is .2 ampere, at a pressure

* By "charging current" is meant the current which flows into the cable when the circuit is open and subjected to normal working pressure.

of 1100 volts and a frequency of 125 p.p.s. What is the capacity of the cable? .2314 microfarad. *Ans.*

Prob. 37. Find the voltage necessary to send 5 amperes through this circuit (Fig. 58). $R = 10$ ohms, $C = 50$ micro-

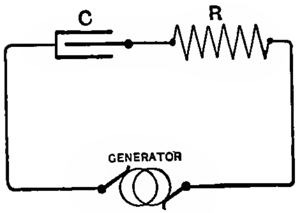


FIG. 58.

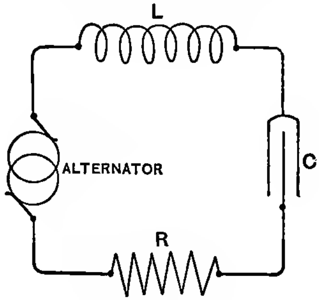


FIG. 59.

farads, frequency = 100 p.p.s. The dynamo resistance is .5 ohm, and its inductance is .05 henry.

$52.535,\ \overline{\diagdown 2^{\circ}\ 15'}$, volts. *Ans.*

Prob. 38. Find the e.m.f. which must be generated by the alternator in Fig. 59 when $C = 5$ microfarads, $R = 3$ ohms, $L = .5$ henry, frequency = 100 p.p.s., current = 5 amperes. The dynamo resistance is .5 ohm, and its inductance is .05 henry. $137.4,\ \underline{\angle 82^{\circ}\ 40'}$, volts. *Ans.*

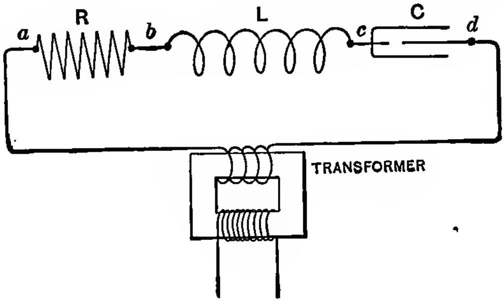


FIG. 60.

Prob. 39. At what frequency would the reactance due to an inductance of .4 henry just neutralize that due to a capacity

of 5 microfarads? The capacity and the inductance are in series. 112.5 p.p.s. *Ans.*

Prob. 40. (Fig. 60.) Given: Frequency = 112.5 p.p.s., e.m.f. at transformer terminals = 100 volts, $R = 10$ ohms, $L = .4$ henry, $C = 5$ microfarads. Find the following quantities: (*a*) Impedance. (*b*) Current. (*c*) E.m.f. *a* to *b*. (*d*) E.m.f. *b* to *c*. (*e*) E.m.f. *c* to *d*. (*f*) E.m.f. *a* to *c*. (*g*) E.m.f. *b* to *d*. Neglect the impedance of the transformer.

(*a*) 10 ohms. *Ans.*
(*b*) 10 amperes. "
(*c*) 100 volts. "
(*d*) $j2828$ " "
(*e*) $-j2828$ " "
(*f*) 2830, $\underline{\angle 87^\circ\, 58'}$, volts. "
(*g*) 0 volts. "

Prob. 41. Find the e.m.f. required to send a current of 10 amperes through the circuit shown in Fig. 61 when the frequency is 60 p.p.s. $L = .3$ henry, $R = 20$ ohms. The dynamo armature resistance is .5 ohm, and its inductance is .05 henry. 298.9, $\underline{\angle 48^\circ\, 13'}$, volts. *Ans.*

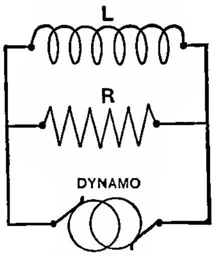


FIG. 61.

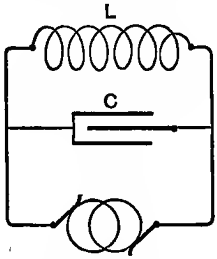


FIG. 62.

Prob. 42. Find the current in C, L, and the main circuit when the pressure is 100 volts (Fig. 62). $C = 10$ microfarads, $L = .05$ henry, frequency = 100 p.p.s. Neglect armature impedance.

Current in C $= j.6283$ amperes. *Ans.*
" " L $= -j3.183$ " "
" " circuit $= -j2.555$ " "

Prob. 43. Find the voltage which would send 10 amperes through the circuit in Fig. 63, which has the following characteristics: Frequency $=$ 60 p.p.s., $L' =$ .05 henry inductance, $L'' =$ .03 henry inductance, $R' =$ 5 ohms resistance, $R'' =$ 7 ohms resistance. Neglect the armature resistance and inductance. 105.46, $\underline{\angle 24^\circ\ 10'}$, volts. *Ans.*

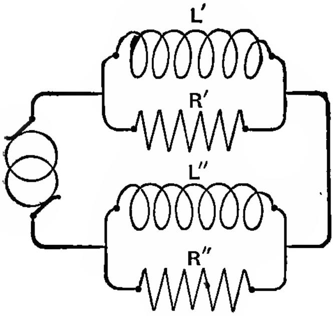


FIG. 63.

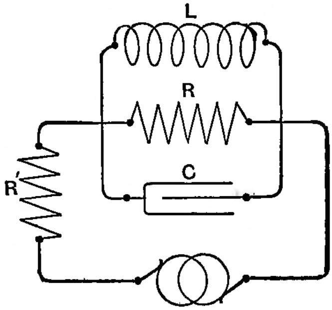


FIG. 64.

Prob. 44. Find the impedance of the circuit shown in Fig. 64, and the phase relations of e.m.f. and current. $R = 5$ ohms, $R' =$ 20 ohms, $L =$ .02 henry, $C =$ 20 microfarads, frequency $=$ 120 p.p.s. Neglect armature impedance.

24.72, $\underline{\angle 2^\circ\ 47'}$, ohms. *Ans.*

Prob. 45. In this circuit (Fig. 65) $R' =$ 50 ohms, $R'' =$ 35 ohms, $L' =$ 1 henry, $L'' =$ .5 henry, $C' =$ 6 microfarads, $C'' =$ 10 microfarads, p.p.s. $=$ 50. Find the total impedance, neglecting armature.

356.2, $\overline{\diagdown 72^\circ\ 42'}$, ohms. *Ans.*

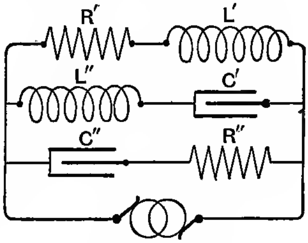


FIG. 65.

Prob. 46. Fig. 66 is a diagram of an electric circuit made

up of several multiple branches and single circuits in series. The constants of this circuit are as follows:

Circuit No. 1.	$R = 100$ ohms,	$L = 2$ henrys.
" " 2.	$R' = 50$ "	$C = 100$ m.f.
" " 3.	$L' = .05$ henry,	$C' = 20$ "
" " 4*a*.	$l = .1$ "	$r = 20$ ohms.
" " 4*b*.	$C'' = 50$ m.f.	$r' = 30$ "

Frequency = 60 p.p.s. Generator circuit, $L_a = .01$ henry, $R_a = 0.5$ ohm. (*a*) What is the impedance of the circuit external to the generator? (*b*) What is the total impedance of the circuit including the generator? (*c*) When the generator delivers 2000 volts to the line terminals of the external circuit, what will be the current in amperes? (*d*) To do this what e.m.f. in volts must the generator develop? (*e*) What phase position will the current occupy with reference to the external or line pressure? (*f*) What will be the phase position of the current with reference to the internal or total generator pressure?

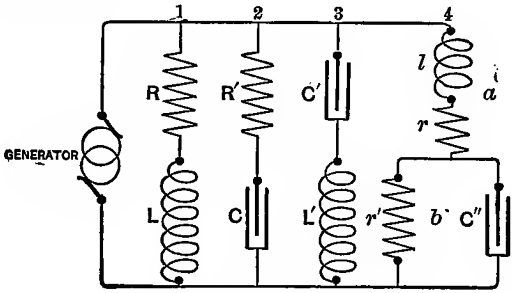


FIG. 66.

(*a*) 29.575, $\overline{\diagdown 9^{\circ}\ 31\tfrac{1}{2}'}$, ohms. ***Ans.***
(*b*) 29.678, $\overline{\diagdown 1^{\circ}\ 32'}$, " "
(*c*) 68.32, $\underline{/9^{\circ}\ 31\tfrac{1}{2}'}$, amperes. "
(*d*) 2027.5, $\overline{\diagdown 1^{\circ}\ 32'}$, volts. "
(*e*) $9^{\circ}\ 31\tfrac{1}{2}'$ ahead. "
(*f*) $1^{\circ}\ 32'$ " "

Prob. 47. In an electric circuit the impressed e.m.f. is

$$E = (190_1 + 60_3 + 30_5) + j(-36_1 + 7_3 + 35_5)$$

and the impedance is

Resistance..........10 ohms

$$\text{Induction reactance} \begin{cases} 5_1 \text{ ohms} \\ 15_3 \quad \text{“} \\ 25_5 \quad \text{“} \end{cases}$$

$$\text{Condenser reactance} \begin{cases} 5_1 \text{ ohms} \\ \dfrac{5}{3}_3 \quad \text{“} \\ 1_5 \quad \text{“} \end{cases}$$

What is the effective value of the current?

Solution:

$$I = \frac{(190_1 + 60_3 + 30_5) + j(-36_1 + 7_3 + 35_5)}{10_1;\ 16.7_3;\ 26.0_5},$$

$$I = \sqrt{19^2 + 3.59^2 + 1.15^2 + 3.6^2 + .419^2 + 1.35^2},$$

$I =$ 19.75 amperes. *Ans.*

CHAPTER VI.

ELECTRIC POWER.

SYNOPSIS.

20. Function of the Electric Circuit.—The function of an electric circuit is to convey electric power exactly as a kinematic chain of connections conveys power from a source to a point of application, except that in the former case the action is molecular, while in the latter there is mass motion. Electric power cannot be used in a circuit as such, but must be transformed into some other state, and this transformation is always accompanied or indicated by the consumption of e.m.f. in the circuit. The sum of all of the consumed e.m.f.s in a circuit is equal to the impressed e.m.f., that is, the e.m.f. furnished by the source of power. As an illustration, take the case of a circuit supplying power to an electric motor. The impressed e.m.f. is consumed in the resistance of the circuit, including that of the motor and dynamo armatures, and in overcoming the counter e.m.f. accompanying the transformation from electrical into mechanical power in the motor, thus accounting for all of the e.m.f. furnished by the generator.

The electric power *which is being transformed in an electric circuit is, at any instant, equal to the product of the e.m.f. and the current.*

This product may be called the instantaneous power to distinguish it from the average power, which is more commonly met with. In order to determine what this average will be in any circuit, the law of the variation of the power must be known and then the average can be determined by a simple integration, either graphical or analytical. In the following paragraphs are discussed a number of special cases, leading up to the general case with which the treatment closes.

21. Case I. Average Power with Sine-form E.m.f. and Current in Phase. (Fig. 67.)

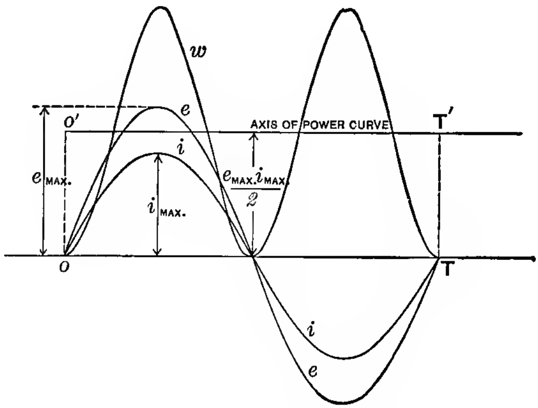


FIG. 67.—Power. Current and E.m.f. in Phase.

Let $e_{\text{max.}} \equiv$ maximum value of e.m.f.;
$i_{\text{max.}} \equiv$ maximum value of current;
$w \equiv$ instantaneous value of power;
$\vartheta \equiv$ angle between the generating vector of the sine curve and the reference axis.

The instantaneous values of the power wave will be the

products of the corresponding instantaneous values of current and e.m.f.

$$w = (e_{\text{max.}} \sin \vartheta)(i_{\text{max.}} \sin \vartheta) = e_{\text{max.}}\, i_{\text{max.}} \sin^2 \vartheta \quad . \quad (49)$$

$$\sin^2 \vartheta = \frac{1 - \cos 2\vartheta}{2}. \quad . \quad . \quad . \quad . \quad (50)$$

Substituting (50) in (49),

$$w = e_{\text{max.}}\, i_{\text{max.}} \frac{1 - \cos 2\vartheta}{2}. \quad . \quad . \quad . \quad (51)$$

By reduction,

$$w = \frac{e_{\text{max.}}\, i_{\text{max.}}}{2} - \frac{e_{\text{max.}}\, i_{\text{max.}} \cos 2\vartheta}{2}. \quad . \quad . \quad (52)$$

The power curve, from equation (52), is a cosine curve of twice the frequency of e.m.f. and current, and it is symmetrical about an axis which is located a distance of $\frac{e_{\text{max.}}\, i_{\text{max.}}}{2}$ above the axis of the e.m.f. and current curves. The average ordinate of this power curve, measured from the axis of the e.m.f. and current curves, is equal to the distance between the two axes. By separating this average ordinate into two factors it is found to be made up of the effective values of e.m.f. and current.

$$\frac{e_{\text{max.}}\, i_{\text{max.}}}{2} = \left(\frac{e_{\text{max.}}}{\sqrt{2}}\right)\left(\frac{i_{\text{max.}}}{\sqrt{2}}\right) = EI\,. \quad . \quad . \quad (53)$$

E and I are effective values of e.m.f. and current. Fig. 67 shows the relations of current, e.m.f., and power curves.

The conclusions reached for the power of sine-form e.m.f.s and currents, *when these are in phase*, may be stated as follows:

1. *The average power is the product of the effective values of e.m.f. and current.*

2. *The power curve is one of twice the frequency of e.m.f. and current and having an axis parallel to that of e.m.f. and current, located above it a distance equal to one half the product of the maximum values of e.m.f. and current.*

22. Case II. Average Power with Sine-form E.m.f. and Current in Quadrature. (Fig. 68.)—As in Case I the instan-

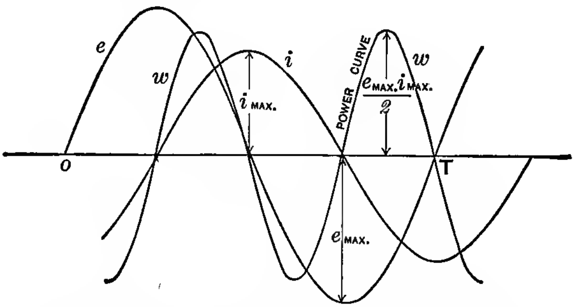


FIG. 68.—Power. Current and E.m.f. in Quadrature.

taneous values of the power curve are the products of the instantaneous values of current and e.m.f.

$$w = (e_{\text{max.}} \sin \vartheta)(i_{\text{max.}} \cos \vartheta) = e_{\text{max.}}\, i_{\text{max.}} \sin \vartheta \cos \vartheta. \quad (54)$$

$$\sin \vartheta \cos \vartheta = \frac{\sin 2\vartheta}{2} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

substituting

$$w = \frac{e_{\text{max.}}\, i_{\text{max.}} \sin 2\vartheta}{2}. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

The power curve from equation (56) is a sine curve of twice the frequency of e.m.f. and current and it is symmetrical about the axis of the e.m.f. and current curves. Its average ordinate, referred to the axis of the e.m.f. and current curves, is zero. Fig. 68 shows the relative phase positions of the components and of the product curves. Conclusion:

The average power transformed in a circuit in which sine-form e.m.f. and current are in quadrature, is zero.

23. Case III. Average Power with Sine-form E.m.f. and Current neither in Phase nor in Quadrature. (Fig. 69.)—As

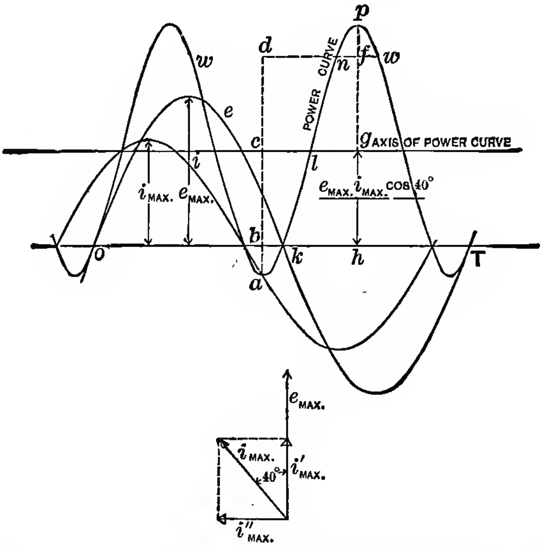


FIG. 69.—Power. Current and E.m.f. not in Phase and not in Quadrature.

before, the instantaneous values of the power are the products of the corresponding e.m.f. and current values

$$w = (e_{\max.} \sin \vartheta)(i_{\max.} \sin (\vartheta \pm \theta)) \quad . \quad . \quad (57)$$

where θ is the phase difference between e.m.f. and current. As θ is an angle always less than 90°, it is evident that Case III is intermediate between Cases I and II, and hence the average product will be between the values found in these extreme cases. As there is a phase difference of θ degrees between e.m.f. and current, it is evident that the current may be resolved into two components, one in phase with the e.m.f. and one in quadrature with it, and that the sum of the average products of these components with the e.m.f. will give the

average product of the original quantities. The component of the current in phase with the e.m.f. is

$$i' = i_{\max.} \cos\theta \sin\vartheta \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

and the component of the current in quadrature with the e.m.f. is

$$i'' = i_{\max.} \sin\theta \sin\vartheta. \quad . \quad . \quad . \quad . \quad . \quad (59)$$

As the quadrature product will be zero, as is shown in Case II, it may be neglected and the average power may be deduced from Case I.

For this component of current,

$$\begin{aligned} w &= e_{\max.} \sin\vartheta\, i_{\max.} \cos\theta \sin\vartheta \\ &= e_{\max.}\, i_{\max.} \sin^2\vartheta \cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (60) \end{aligned}$$

but

$$\sin^2\vartheta = \frac{1 - \cos 2\vartheta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

$$w = e_{\max.}\, i_{\max.} \frac{1 - \cos 2\vartheta}{2} \cos\theta \quad . \quad . \quad . \quad . \quad (62)$$

$$w = EI \cos\theta \text{ (see case I)}. \quad . \quad . \quad . \quad . \quad . \quad (63)$$

Stated at length, the formula just deduced becomes:

The power transformed in any circuit in which sine-form e.m.f. and current are present is the product of the effective values of e.m.f. and current and the cosine of the angle of phase difference between them.

To the product of e.m.f. and current in such a circuit is given the name **apparent power** to distinguish it from the **real power** given in Case III. The ratio of the *real* to the *apparent* power is given the name **power factor,** which is therefore equal to the cosine of the angle of phase difference between current and e.m.f. The apparent power may be considered as com-

posed of two factors known as the real power and the **wattless component,** sometimes called the **wattless power.** The former of these is equal to

$$EI \cos \theta,$$

the latter being

$$EI \sin \theta.$$

The term $\cos \theta$ in the power equation

$$Power = w = EI \cos \theta$$

is called the **power factor**, and the term $\sin \theta$ in the equation

$$wattless\ component = EI \sin \theta$$

is called the **wattless factor.**

In circuits containing only reactance and resistance, and therefore those in which all of the power is transformed into heat, a number of interesting relations hold, as follows:

Let $r \equiv$ resistance;

$x \equiv$ reactance;

$z \equiv$ impedance;

$j \equiv$ symbol indicating the quadrature relation of the term following it to those terms not preceded by it.

$$I = \frac{E}{z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Expressing current in equation (57) in its two quadrature components with respect to e.m.f.,

$$I = I\frac{r}{z} + jI\frac{x}{z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Multiplying through by e.m.f.,

$$EI = EI\frac{r}{z} + jEI\frac{x}{z}. \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Since $EI\frac{x}{z}$ is in quadrature with the e.m.f., it represents no power, so that the average power transformed in the circuit into heat is,

$$W = EI\frac{r}{z} = EI\cos\theta. \quad . \quad . \quad . \quad . \quad (67)$$

Illustrative Problem.—A sine-form of e.m.f. of a maximum value of 1557 volts when impressed upon a certain circuit sets up therein a current of a maximum value of 14.1 amperes. A wattmeter reading average watts placed in the circuit shows a power consumption of 10 k.w. What is the power factor of the circuit? 91.1%. *Ans.*

24. Case IV. Average Power with Non-sine-form e.m.f. and Current.—The harmonic component method of analysis finds an application in determining the power transformed in a circuit in which a non-sine-form e.m.f. establishes a non-sine-form current. Since alternating values with different frequencies produce an average product of zero, and since the power in an alternating current circuit is the average of the products of the instantaneous values of e.m.f. and current, it follows that the power is equal to the sum of the products of the effective values of the harmonic e.m.f. and current components and their respective power factors.

$$W = e_1 i_1 \cos\theta_1 + e_3 i_3 \cos\theta_3 + e_5 i_5 \cos\theta_5 + \dots \quad . \quad (68)$$

In a circuit where all of the power is lost as heat this equation may be written

$$W = E_1 I_1 \frac{r}{z} + E_3 I_3 \frac{r}{z_3} + E_5 I_5 \frac{r}{z_5} \dots E_n I_n \frac{r}{z_n}. \quad . \quad (69)$$

As a general conclusion it should be stated that Case III covers all the requirements of practice except where an

analysis of the form of the power curve is desired. As all measuring instruments are calibrated in terms of effective current and e.m.f., it follows that irregular current and e.m.f. curves can be considered as replaced by equivalent sine curves, in which case the conclusions drawn in Case III are capable of general application. It should be noted also that the common form of e.m.f. and current curves does not differ greatly from that of the sine curve.

25. The Equivalent Sine Wave.—An inspection of problem 47 shows that where the impressed e.m.f. in a circuit does not have a simple sine-form and is therefore irregular the current form is likewise irregular. The effective values of the e.m.f. and current will be

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2 + \dots} \quad . \quad . \quad . \quad (70)$$

and

$$I = \sqrt{I_1^2 + I_3^2 + I_5^2 + \dots} \quad . \quad . \quad . \quad . \quad (71)$$

These values are indicated directly by any properly constructed and calibrated alternating pressure or current instrument. Likewise any corresponding wattmeter placed in the circuit indicates directly the power in watts. Thus for the general alternating current circuit in practice three out of the four quantities in power equation,

$$W = EI \; (\textit{Power Factor}) \quad . \quad . \quad . \quad . \quad (72)$$

are easily determined, and the fourth, *power factor*, equals

$$\textit{Power Factor} = \frac{W}{EI}. \quad . \quad . \quad . \quad . \quad (73)$$

In the general case

$$\frac{W}{EI} \textit{ does not actually equal } \cos\theta.$$

In fact there is no definite value for θ with which to express the angle of phase difference of two irregular waves. Such irregular waves are made up of a number of components wherein corresponding components have different angles of phase differences and frequencies. See Section 13. No one phase difference can, strictly speaking, be applied to a pair of aggregations of such components.

Practical requirements demand, however, the convenience of an equivalent angle of phase difference in lieu of a multi-valued or composite angle difficult of expression or even of comprehension. Thus the custom has been formed of making in the general case

$$\textit{Power Factor} = \cos\theta.$$

From this it follows that the power equation for the general case wherein the e.m.f. and current may have any form must be written

$$W = EI\cos\theta,$$

an equation that must be interpreted only as follows:

$$\cos\theta = \frac{W}{EI},$$

where E, I, and W have been determined by alternating current, pressure, and power instruments giving the values

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots}$$
$$I = \sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots}$$
$$W = IE.$$

With this conventional meaning of $\cos\theta$ a corresponding conventional meaning must be attached to effective values of current and e.m.f., I, and E. This convention consists in considering I and E to be **equivalent sine waves** of current and e.m.f. having effective values equal to the effective values of the actual irregular waves. The significance of equivalent sine wave may now be made clearer by reference to Fig. 70,

which illustrates irregular waves of e.m.f. and current and their equivalent sine waves.

In Fig. 70 the irregular wave of e.m.f., E, established the irregular wave of current, I, differing in form from that of E. It is required that the equivalent sine waves of E and I be

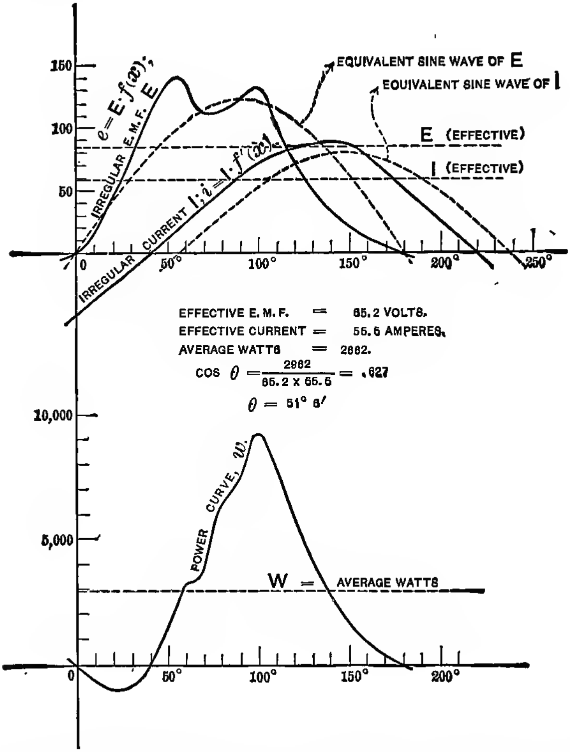


FIG. 70.—Illustrating the Meaning and Application of Equivalent Sine Waves.

determined in amount and phase position and that such waves be drawn to scale in this diagram so that they may be compared with their corresponding irregular waves.

The instantaneous product of these curves produces the power curve having a form and average value as drawn. The

effective values of E and I are also drawn in the diagram. They are determined by the method given in Section 13, p. 57.

For equivalent sine waves

$$\textit{Power Factor} = \cos\theta = \frac{W}{EI}.$$

The equivalent sine waves are now determined after assuming that the phase position of the current I is to be referred to that of E. The values from point to point of the equivalent sine waves then are

$$e = \sqrt{2}E \sin\phi.$$

$$i = \sqrt{2}I \sin(\phi + \theta).$$

The corresponding curves have been platted point by point and then drawn in in full in Fig. 70.

If the irregular current wave had been chosen as the one to which the phase of the wave of e.m.f. is to be referred the equivalent sine wave of current would pass through zero in a positive direction at the same instant that the irregular wave of current passes through zero in the same direction. From the diagram it is seen that this would result in shifting the phases of the equivalent sine waves of current and e.m.f. backward 16 degrees with respect to their corresponding irregular waves and with respect to the position at present determined for them in the diagram.

Thus the precise meaning and significance of the equivalent sine wave are made clear. In most routine practice E, I, and W as read from the instruments and $\cos\theta$ as deduced are interpreted for convenience as equivalent sine waves and the equivalent angle of phase difference wherever the actual waves E and I are irregular.

In all accurate testing, research, and study the equivalent sine wave leads to error and confusion.

The following problem will illustrate the solution of a power problem in irregular wave forms by means of harmonic component EI products, eliminating the use of equivalent sine waves:

Prob. 48. In an electric circuit transmitting power the generator produces an impressed e.m.f. of

$$E' = (1900_1 + 600_3 + 300_5) + j(-360_1 + 70_3 + 350_5)$$

and the motor which disposes of the power transmitted by this circuit produces a counter-e.m.f. of

$$E'' = + (1600_1 - 200_3 + 100_5) + j(-60_1 + 100_3 + 50_5).$$

The impedance of the total circuit is

$$\text{Resistance} \ldots\ldots\ldots \ 3 \text{ ohms}$$

$$\text{Induction reactance} \begin{cases} 2_1 & \text{"} \\ 6_3 & \text{"} \\ 10_5 & \text{"} \end{cases}$$

Required the electric power developed by the generator.

$$I = \frac{E' - E''}{Z} = \frac{(300_1 + 800_3 + 200_5) + j(-300_1 - 30_3 + 300_5)}{3 + j(2_1,\ 6_3,\ 10_5)},$$

$$I = (23.1_1 + 49.3_3 + 33.0_5) - j(115.4_1 + 108.7_3 + 10.1_5).$$

Power equals

$$W = I_1E_1' + I_3E_3' + \ldots$$

$$W = [(1900_1 + 600_3 + 300_5) + j(-360_1 + 70_3 + 350_5)] \times$$
$$[(23.1_1 + 49.3_3 + 33_5) - j(115.4_1 + 108.7_3 + 10.1_5)]$$

$W = 113{,}680$ watts. *Ans.*

CHAPTER VII.

MAGNETOMOTIVE FORCE AND THE LAWS OF THE MAGNETIC CIRCUIT.

26. Magnetomotive Force and the Magnetization Curve.—*a. The Unit of Magnetomotive Force, M.m.f., the Gilbert. Symbol H.*—Magnetomotive force is the cause which results

in the production of magnetic flux. By definition *one unit of magnetomotive force will establish in air a unit density of magnetic flux through a distance of unity.* The name of this unit is the **gilbert.*** The practical and *c.g.s.* units in this case are identical. The definition of the gilbert is illustrated in Fig. 71. Over the cross-section of one cube of air and throughout

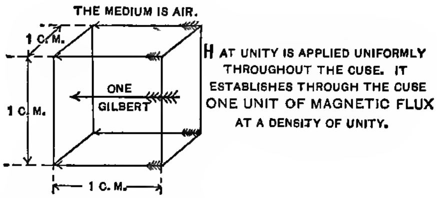


FIG. 71.—Graphical Illustration of Definition of Gilbert.

its length the application of a m.m.f. of one gilbert will establish through the cube one maxwell of flux uniformly distributed over the cross-section of the cube at a density of unity. See definition of maxwell, Section 5, page 13.

b. The Magnetomotive Force of the Ampere-turn.—In Fig. 72 the straight conductor *CC* carries a current which, when measured in *c.g.s.* units, equals *I*. From experimental researches it has been found that when such a straight conductor carries current, a field of magnetic flux is set up about it. When the conductor is remote from magnetic bodies, the flux is everywhere at right angles to the conductor, its density varies inversely as the normal distance from the conductor, and closed circuits of equal flux density are circular about the conductor and, therefore, everywhere equidistant from it. Since the flux density at any point is inversely proportional to the distance of the point from the conductor, the

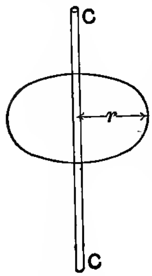


FIG. 72.

* Names of units. Action of the American Institute of Electrical Engineers Transactions, Vol. XI, 1894, p. 48.

m.m.f. per *cm.* which sets up the flux at this point is also inversely proportional to the distance from the conductor. As the length of any circular flux circuit is *directly* proportional to its distance from the conductor, and as the m.m.f. per *cm.* is *inversely* proportional to this distance, it follows that the total m.m.f. established in any complete flux circuit is constant for a given current flowing in the conductor.

The m.m.f. around a straight conductor of infinite length which carries a current of one ampere is $\frac{4\pi}{10}$ *gilberts.*

The value of

$$k = \frac{4\pi}{10}$$

is derived in the following manner:

In any space not actually occupied by magnetic bodies the mechanical tension of magnetic flux along its own direction is

$$\frac{B^2}{8\pi}$$

dynes per *sq. cm.* (See Sec. 4.) No tension existed in the molecular magnetic mechanism before the magnetic flux came into existence. The conservation of energy requires, then, that work be done when magnetic flux is established. It requires that the amount of this work shall be equal to the product of the tension into the distance through which the tension is produced. In the field about a conductor carrying current, as in Fig. 72, the *mechanical expression* for the work done in establishing a small filament of magnetic flux through any concentric circuit about the conductor will, therefore, be

$$w = 2\pi \cdot r \cdot \frac{B^2}{8\pi} \cdot A, \quad . \quad . \quad . \quad . \quad . \quad (74)$$

where

$$\frac{B^2}{8\pi}$$

is the intensity of flux tension, $2\pi r$ the distance through which it has been established, and A is the area of the cross-section of the filament. An *electrical expression* for the work done in establishing this same flux filament may also be written. During the process of establishing the current in the conductor, the flux about the conductor was being formed by the m.m.f. of the current. This caused a counter-e.m.f. to be generated in the conductor which was equal in amount to the rate of increase of the magnetic flux in the filament. The rate at which work is done in establishing the magnetic field is, at any instant, the product of the current and the counter-e.m.f. It follows, then, that when the formation of a magnetic field about a conductor by the current in the conductor is in progress, electrical power is absorbed from the circuit equal at each instant to the product of the current into the counter-e.m.f. The energy of this power disappears as potential energy in the formation of the field of magnetic flux. When the current whose m.m.f. maintains the field of flux is brought to zero, the field of flux likewise comes to zero, and in so doing its potential energy is given back to the electric circuit by the formation of an e.m.f. in the circuit. This has the same sign as the current, and it exerts itself to maintain the current for a time when the supply pressure is cut off.

The energy thus taken from the circuit or restored to it again by the magnetic field is a measure of the work done in bringing the magnetic field into existence.

At any instant

$$dw = EIdt. \qquad (75)$$

Since flux density varies inversely with distance from the conductor,

$$B = \frac{kI}{2\pi r},$$

or

$$I = \frac{2\pi r}{k} B,$$

where k is the m.m.f. about a unit current in a straight conductor. The counter-e.m.f., due to the changing flux in the filament under consideration, is

$$E = \frac{d\Phi}{dt}, \quad \ldots \ldots \ldots \quad (76)$$

where $\Phi = BA$, or the maxwells in the filamentary magnetic circuit. Substituting these quantities in the equation

$$dw = IEdt, \quad \ldots \ldots \ldots \quad (77)$$

we have

$$dw = \frac{2\pi r}{k} \cdot B \cdot A \cdot dB, \quad \ldots \ldots \ldots \quad (78)$$

$$w = \frac{2\pi r}{k} \cdot A \cdot \int BdB = \frac{2\pi r}{k} \cdot A \cdot \tfrac{1}{2} B^2. \quad \ldots \quad (79)$$

Equating (74) and (79),

$$2\pi r \cdot \frac{B^2}{8\pi} \cdot A = \frac{2\pi r}{k} \cdot A \cdot \tfrac{1}{2} B^2,$$

and solving for k,

$$k = 4\pi.$$

Therefore 4π equals the number of gilberts of m.m.f. exerted in any concentric circuit about a conductor carrying one *c.g.s.* unit of the current. Since the value of one ampere is one tenth of the value of the *c.g.s.* unit it follows that *the m.m.f., exerted in any complete concentric circuit about a conductor carrying one ampere of current is* $\frac{4\pi}{10}$, *or 1.257 gilberts.*

This statement is true whether the path be in a plane normal to the conductor or not.

Proof: If the path does not lie in a path perpendicular to the conductor, it may be divided into elemental steps, alternately in this plane, and perpendicular to it or parallel to the conductor. Along the latter steps the m.m.f. is zero. An equivalent path may therefore be chosen which is in a plane perpendicular to the conductor. This path in the limit would be made up of indefinitely small segments lying alternately in arcs of circles and in their corresponding radii. The m.m.f. applied along such radii would evidently be zero. Let *pqstp*, Fig. 73, be such a path, linking with the conductor *i*. Let *aboda* be a circular path in the same plane, and concentric with the conductor. The path *pqstp* may be divided into steps composed of circular arcs and radial distances. Along the latter steps the m.m.f. is zero. Along each of the former the m.m.f. (see p. 107) is equal to the m.m.f. along an arc of the circle *aboda* which is included between the same radii.

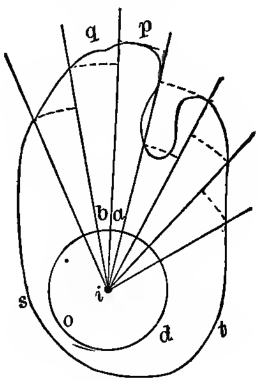


FIG. 73.

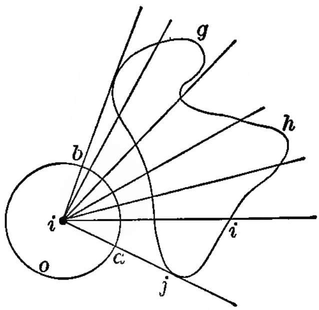


FIG. 74.

In going completely around the path *pqstp* the sum of the m.m.f.s found is therefore equal to the sum of all the elemental m.m.f.s around the circumference of the circle, each taken once, or if more than once, the excess being half of one sign

and half of the opposite sign. The m.m.f. around the irregular path is therefore equal to the m.m.f. around the circle, and is

$$\frac{4\pi}{10}I,$$

where I is the current in amperes.

The m.m.f. due to any current, taken completely around a closed path which does not link with the current, is zero.

Proof: In Fig. 74, the m.m.f. taken completely around the path *ghij* includes components making a m.m.f. equal to that from *a* to *b*. But each component is included an even number of times, half positive and half negative, thus giving a total sum of zero.

c. The Conductor-turn, Helix, and Solenoid.—If the conductor be bent so as to form a single circular turn, as in Fig. 75, the m.m.f. due to the current in the turn of conductor will be present everywhere around the conductor. Under these circumstances the field of flux that will be established is represented in Fig. 76. The lines about the conductor show the direction of the flux at all points about the turn, while their proximity is proportional to the flux density, or B. The

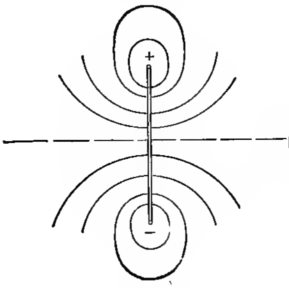


FIG. 76.

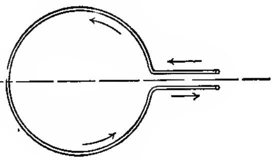


FIG. 75.

m.m.f. applied by the current in the conductor-turn is everywhere alike for any closed circuit passing through the turn and, therefore, around the conductor. The variable nature of

the flux as to amount and direction is due to the varying cross-section of the space about the conductor which at the minimum portion is limited to the cross-section enclosed by the turn, and at the maximum portion is the unlimited cross-section of indefinite space. These two facts account entirely for the varying field of flux as given in Fig. 76.

If the conductor be coiled into a helix, as in Fig. 77, it is found that the character of field established around the conductor by the current in it is given by the lines in Fig. 78. In this case, as in that of a single turn of the electric circuit, it is found that the m.m.f. is the same for any closed route through the helix and, therefore, around the conductor, and that the amount and direction of the flux is determined by the variable length and character of the magnetic flux circuit. There is this difference, however, to be observed. In the case

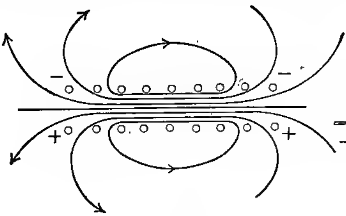


FIG. 78.

FIG. 77.

of a single turn any closed circuit through the turn passes around the conductor once, *while for the helix the closed flux circuit through the helix passes around the conductor as many times as there are turns in the helix.* The m.m.f. exerted by a helix along any closed circuit through it is greater than the corresponding m.m.f. of a single conductor by as many times as there are turns in the helix.

When a conductor spiral is made up of more than one layer of turns it is technically called a *solenoid*, though generally in practice it is called simply a *coil.* The solenoid or coil has the same m.m.f. properties as those of the helix.

In electrical engineering, the chief and practically the only source of m.m.f. is the current-conducting solenoid or coil. Its m.m.f. equals the product of its turns and current, times 1.257. Thus in a solenoid having n turns carrying I amperes, the m.m.f. $= 1.257nI$. *The product* nI *in practice is called* **ampere-turns.**

d. Magnetic Reluctance. Its Unit.—Magnetic reluctance, or more generally **reluctance**, is the property of every magnetizable region whereby the amount of magnetic flux established by a given amount of m.m.f. is limited. It is analogous to resistance in the electric circuit.

One unit of reluctance in a magnetic circuit will require one gilbert to establish one maxwell.

The reluctance across a cube, one centimetre on a side, is

$$\frac{H}{B} = \rho.$$

ρ is therefore the symbol for *specific reluctance.*

e. Magnetic Permeability.—Magnetic permeability, or more generally **permeability,** is the reciprocal of specific reluctance and is, therefore, analogous to specific conductivity in the electric circuit. Its value is

$$\mu = \frac{B}{H}.$$

f. The Magnetic Circuit.—With the exception of iron, nickel, and cobalt, no substance or material used in engineering when brought into the magnetic circuit will modify the character of the flux in amount and direction appreciably. On the other hand, when iron, nickel, or cobalt is placed in the magnetic circuit a powerful modification in the resulting flux occurs, both in amount and direction. Thus if a bar of soft iron be placed in a solenoid carrying a strong current, the bar at once becomes a powerful magnet. As soon as the current in the solenoid ceases the bar of soft iron will cease to

be a magnet. In Fig. 79, if the bar *NS* as well as the keeper *AAA* be made of soft wrought iron and a solenoid surrounds *AAA*, the effects produced by introducing such materials into the magnetic circuit of a solenoid may be accurately observed. The arrangement of the solenoid through which passes the circuit occupied by the soft wrought-iron bars *AAA* and *NS* is shown in Fig. 79. By weighing the tension of the flux

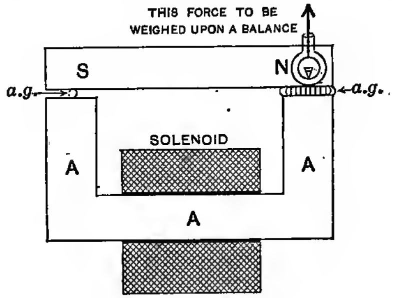


FIG. 79.

established across *ag*, data will be obtained showing the properties of the soft wrought iron in the magnetic circuit. Assume that sufficient current, *I*, is passed through the solenoid to produce a flux tension at *ag* which may be weighed on a balance.

The flux produced is set up in closed circuits which pass through the solenoid. It has already been shown that the m.m.f. due to the solenoid, taken around any particular circuit, is the same as that around any other. The flux set up around any path by this constant m.m.f. is therefore *inversely proportional to the length of the path, and directly proportional to the area of its cross-section.* These quantities remaining the same, *the flux depends upon the character of the material through which it passes*, i.e., upon its permeability.

Example: In Fig. 79 the permeability of the iron parts *AAA* and *NS* is so high that it is quite accurate to assume that *all* the flux follows the iron path where possible, and goes straight from iron to iron at their gaps. The flux crossing the

air-gaps is then equal to the induction through the iron, and the area over which this flux is distributed is sensibly equal to the area of the *end* of the bar AAA, the distance across the air-gap being made very small.

Let P be the observed weight in dynes, A the area of cross-section in *sq. cms.* at ag, and l the distance in *cms.* through both air-gaps, ag; then

$$\frac{P}{A} = \frac{B^2}{8\pi}$$

and

$$B = \left(\frac{8\pi P}{A}\right)^{\frac{1}{2}}. \qquad \qquad (80)$$

The portion of the m.m.f. of the solenoid that is used in putting the flux across the air-gaps equals

$$H = Bl \textit{ gilberts}.$$

If the number of turns in the solenoid be n, then the m.m.f. of the solenoid in this experiment is nI ampere-turns, or $1.257nI$ gilberts. Now if the value of B, as determined by the balance, were found to be between zero and 12,000 maxwells per *sq. cm.*, the m.m.f. used at the air-gaps will be found to be a large part of the total m.m.f. of the solenoid, $1.257nI$. This means that the iron part of the circuit, at flux densities below 12,000, consumes but little of the m.m.f. of the solenoid, while the remainder, or most of this m.m.f., is consumed at the air-gaps.

Should the current, I, be increased to I_2 in the solenoid, B at ag will be increased to a larger value, B_2. If in this way I_2 has been made large enough to cause B_2 to attain a value above 14,000 maxwells per *sq. cm.*, the m.m.f. consumed by the iron part of the magnetic circuit,

$$\text{m.m.f.}_1 = 1.257nI_2 - lB_2, \qquad \qquad (81)$$

will be much greater in proportion than when the flux density was less, or B. In fact, for values of B above 14,000 the soft

wrought iron will suddenly increase its reluctance from a very low value to a value nearly as high as that of air and ordinary materials.

All those materials which exhibit this low reluctance for magnetic flux at moderate densities are called **magnetic materials.**

g. Magnetization Curves.—To make a close study of the properties of a magnetic material, the magnetic balance may be used. The experiments with soft wrought iron in the magnetic balance in the preceding sub-section *f* should be carefully made. Small air-gaps at *ag* and *ag* are used to eliminate the error due to the fringes of flux over the edges. Frequent weighings of the flux tension and current readings are made at intervals corresponding to approximately uniform increments of current. The corresponding values of B and H for the iron part of the circuit are deduced from equations (80) and (81). In (81) the values of H are divided by the length in centimetres of the iron part of the magnetic circuit, thus giving the m.m.f.s consumed in the iron per centimetre length. These values for soft electrical sheet steel are platted with rectangular coördinates in Fig. 80, forming the "*sheet steel*" curve there drawn.* A similar curve for the best qualities of soft wrought iron is almost identical with the one here drawn for electrical steel. Such a graphic representation of the properties of a magnetic material is called a *B-H curve.* It shows clearly that the steel has but small reluctance to the establishment of magnetic flux at densities below $B = 80{,}000$

* The *B-H* curves in Fig. 80 are platted in the (*maxwells per square inch*)-(*ampere-turns per inch*) system. For engineering purposes in the United States it is now customary to use ampere-turns per inch and maxwells per square inch for *B-H* values in lieu of gilberts per centimetre and maxwells per square centimetre. The *B-H* values in Fig. 80 may be reduced to the *c.g.s.* system, i.e., to the gilbert-maxwell-centimetre system of values, by dividing the B values by 6.45, the number of square centimetres in a square inch, and the H values by $(2.54 \div 1.257)$, the centimetres in an inch and the gilberts in an ampere-turn.

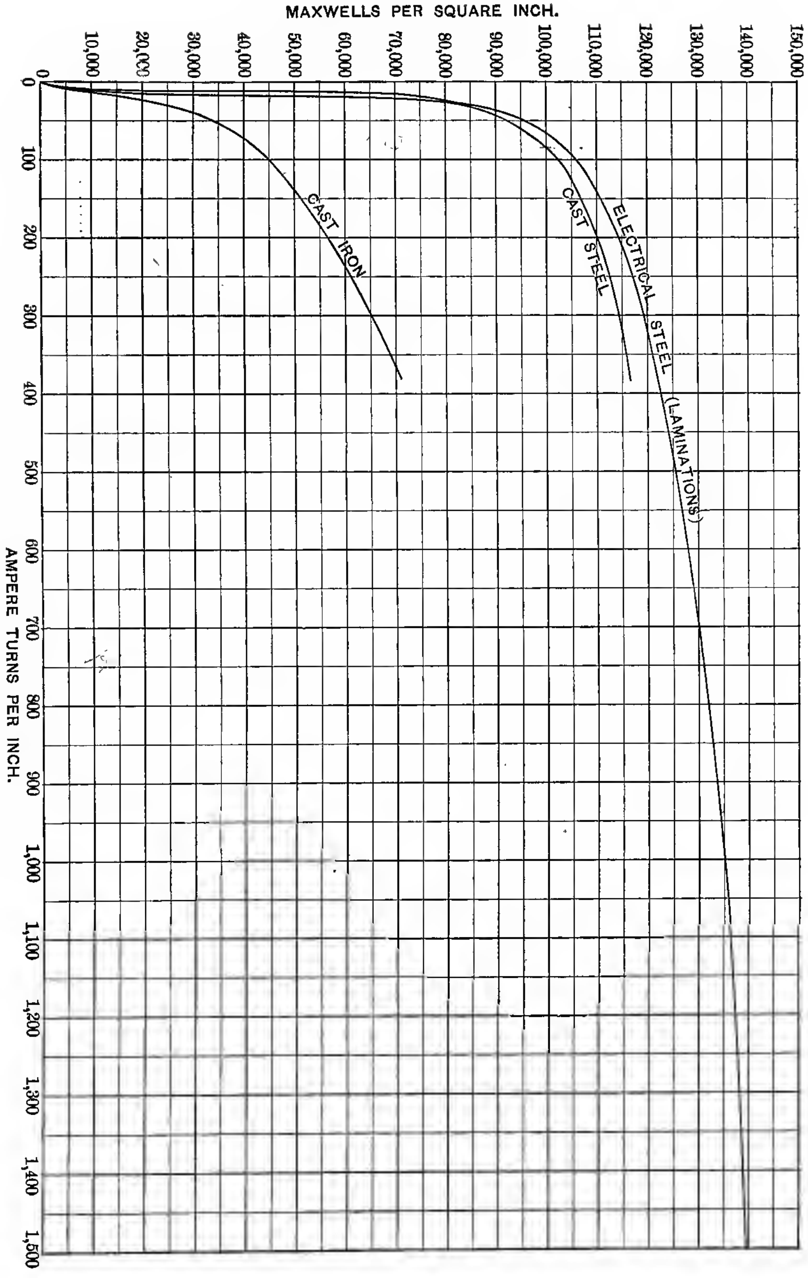


FIG. 80.—Magnetic Properties of Irons.

per sq. in. Above 80,000 per sq. in. the m.m.f. consumed per inch length increases much more rapidly in proportion to the increase in the flux density, showing that the reluctance in the neighborhood of 80,000 maxwells per sq. in. no longer remains uniformly small. For values above 80,000 the reluctance rapidly increases up to the value $B = 105{,}000$, after which there is but slight farther increase in the reluctance.

The B-H curve above this point approaches more and more nearly to a straight line. This line ultimately becomes approximately parallel to a straight line which may be drawn through the origin, representing the relation between the magnetic flux, B_{air}, in maxwells per sq. in., established in air, and the magnetizing force in ampere turns per inch length, which is necessary to set up the flux, B, in the steel or iron. This relation is $H = B_{\text{air}}$, from definition, *c.g.s.*, and the equation of this part of the curve is approximately

$$B = 3.19\, H + C,$$

where C is a constant representing the distance this part of the B-H curve lies above the H-B_{air} curve, or it represents the increased induction due to the presence of the iron before it becomes *saturated.*

h. SATURATION.

A magnetic material is said to be saturated when it already carries such a dense induction that a further increase in the magnetizing force, H, produces no more increase in the induction than would be produced in air by a similar increase in magnetizing force.

To reach this limit absolutely requires in iron an extremely high magnetizing force, between 2000 and 5000 gilberts per centimetre length, or from 4000 to 10,000 ampere-turns per inch of length. Iron may be very nearly saturated, however, by much smaller forces. The so-called "*knee*" of the B-H

curve is found in Fig. 80 at about $H = 40$. This is a region of "*approaching saturation*," and for practical purposes the iron may be considered saturated above this point, as large increases in H are thereafter necessary for comparatively small increases in B. It is usual to speak of the iron when in this condition as **saturated**; meaning thereby not that the actual point of saturation as defined has been reached or passed, but that an abrupt change in its magnetic susceptibility has occurred, and that thereafter its behavior is very much as if it were in fact saturated.

27. Matters Affecting Permeability.—*a. Permeability.* The symbol μ is used for permeability.

Then $B = \mu H$, or density of magnetic induction is the product of permeability and magnetizing force.

Evidently if μ is a constant for any material, the B-H curve is a straight line for that material. Conversely, also, if the B-H curve for any material is not a straight line, μ for that material is not constant. For air and for all materials except iron, nickel, and cobalt, μ is very nearly unity. The relations between μ and H for the "sheet steel" curve, and for the "cast iron" curve of Fig. 80, are given in Fig. 81.

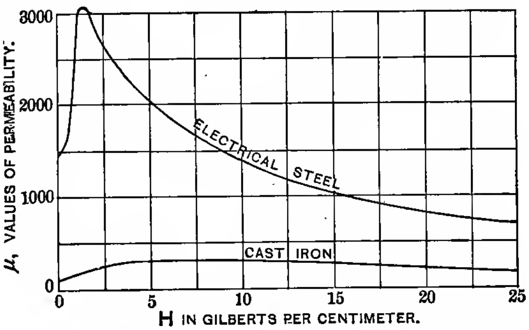


FIG. 81.—Permeability Curves for Electrical Sheet Steel and Cast Iron.

It will be noted that the permeability is small at low densities of magnetization, that it rises rapidly to a maximum, then falls

rapidly to a value from which it gradually diminishes indefinitely. In Fig. 81*a* the μ-H relation is given for wrought iron

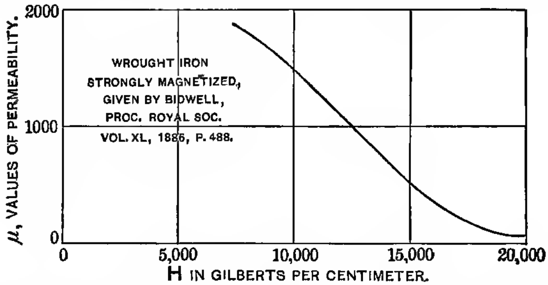


FIG. 81*a*.—Permeability Curve for Wrought Iron Strongly Magnetized.

as obtained by Bidwell at very high flux densities established by corresponding high values of H applied per centimetre length in the iron.

b. Effect of Temperature on Permeability.—The permeability of iron at ordinary temperatures varies but little with change in temperature. Very remarkable effects are found at high temperatures. The curve in Fig. 82, given by Hopkinson, shows the relation between permeability and temperature from 0° to 800° C.* Curve A was taken with a magnetizing

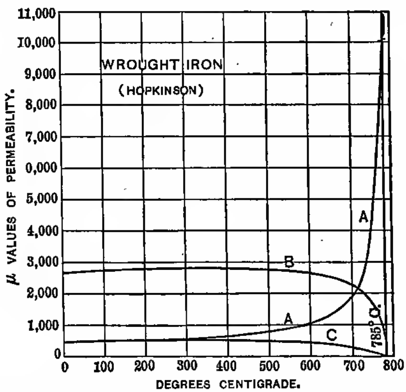


FIG. 82.—Effect of Temperature upon the Permeability of Wrought Iron.

* Hopkinson, *Electrician*, Vol. XXIV, p. 245, January 10, 1890.

force of $H = .3$, curve B with $H = 4$, and curve C with $H = 45$, where $H =$ gilberts per centimetre length of the wrought iron. In each case the permeability falls to about unity at a temperature of 785. With the two larger magnetizing forces the change in permeability is rather abrupt, but with the smaller value of H the suddenness of this change is much more marked, and immediately before falling the permeability rises very rapidly to the extremely high value of 11,000. The point at which this change takes place is one where other changes in the physical properties of the iron occur. If a rod of iron is heated to a bright red and allowed to cool slowly, it is noticed that at this "*critical temperature*" there is a sudden check, for a brief period, to the cooling; in some forms of iron this is so marked as to produce a noticeable brightening in its redness. This phenomenon has been given the name of "*recalescence.*" At the same time the rod, which has previously been contracting, lengthens for an instant before continuing to shorten.

c. Effect of Physical Treatment on Permeability.—The permeability of iron or steel is very largely dependent on the physical treatment which it has received. In general the softer grades are more permeable, and any process tending toward softening, such as annealing, increases the permeability. Any process of hardening decreases the permeability. The curves A and B, Fig. 83, show the effect of annealing on permeability. Curve A is taken from an ordinary sample of electrical steel casting, and curve B, from the same sample after thorough annealing.

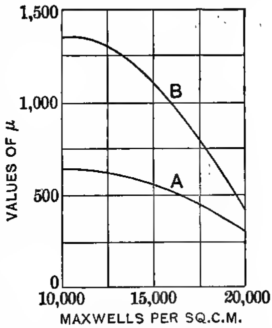


FIG. 83.—The Effect of Anneal ing upon the Permeability of Electrical Steel Casting.

d. Effect of Impurities on Permeability.—Almost all im-

purities in iron have a detrimental effect upon its permeability. Some are of a much more marked character than others. Combined carbon is particularly bad. Manganese, nickel, chromium, tungsten, silicon, phosphorus, and sulphur, when present in more than ordinarily small amounts, are also injurious. So many things operate to affect the permeability of a magnetic substance that it is difficult to assign the proper weight to each. It has been found that many substances may be present in iron or steel in small amounts as impurities without appreciably affecting its permeability. Again two impurities will often offset the detrimental characteristics imparted by either. The mechanical character of irons and steels is also improved or harmed by the presence of impurities. The presence of certain impurities is often necessary to impart to the iron or steel the best mechanical-physical characteristics. The engineer, therefore, who undertakes to produce electrical steels and irons must make a judicious use of the impurities so as to produce a good foundry and machine-shop material and yet have it suffer least in permeability. Modern manufacturers are able to produce grades of "electrical" iron and steel which are fairly uniform and highly permeable. It is not possible, however, to predict the permeability of a steel, or iron, exactly, in advance of a test, and different samples from the same lot of iron, or even from different parts of the same sheet, may show appreciable differences in magnetic quality.*

28. Reluctance of the Magnetic Circuit.—*a. Reluctance.* —The counter action of any region to the setting up of magnetic flux through it has been termed its **reluctance**. *Specific reluctance* is essentially the reciprocal of permeability and its symbol is

$$\rho = \frac{1}{\mu}.$$

* An excellent paper on this subject by Parshall and Hobart is to be found in *Engineering*, London, January 21, 1898.

The specific reluctance for any material is the magnetic reluctance of a cube of that material one *cm.* on a side. We may write as an expression for the magnetic circuit which is similar to Ohm's law for the electric circuit

$$Flux = \frac{M.m.f.}{Reluctance}.$$

The actual reluctance along any path is inversely proportional to its area and directly proportional to its length. Thus the reluctance of a bar of length l and cross-section A is

$$Reluctance = \frac{l}{\mu A} \quad . \quad . \quad . \quad . \quad . \quad (82)$$

and the expression for the magnetic circuit becomes

$$Flux = \Phi = m.m.f. \frac{\mu A}{l} \quad . \quad . \quad . \quad . \quad (83)$$

which is a practical working expression. $\frac{BA}{l}$

In the solution of an actual problem, it is necessary to know, or to assume, an approximate value for the flux density, as the value of μ depends upon this.

b. Comparison of Magnetic and Electric Circuits.—A magnetic circuit to which the relations just given may be applied, differs from the electric circuit, for which a similar expression has been used, in one important particular. The electric circuit is confined to a metal path or conductor. The conductor is surrounded by insulating materials which prevent the escape of current. On the other hand, no substance is an insulator for magnetic flux. The poorest conductor of magnetism has a permeability little less than unity. It is, therefore, impossible to confine the flux to any definite path. We may, indeed, provide a path of such low reluctance that the greater part of the flux will be found in it, but a field of greater or less intensity, and extending to an indefinite distance, will surround this path.

In Fig. 84 is shown the magnetic circuit of an early and familiar type of two-pole dynamo. The useful magnetic flux

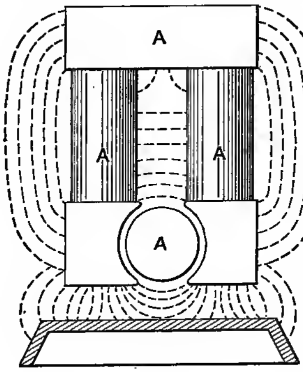


FIG. 84.

lies in the path, *AAAA*, but surrounding this path is a "*leakage*" or "stray" field, which in this type of machine often amounts to 40 per cent of the whole.

c. Amount of Magnetic Leakage.—It is often necessary to estimate the amount of leakage which will occur in air between iron surfaces, when a m.m.f. acts across the intervening space. It is only possible to do this approximately at best, and it is, therefore, not desirable to enter into an elaborate discussion of this matter. A few simple cases will serve as illustrations. The reluctance across any path in air is its average length in centimetres divided by the average area of its cross-section,

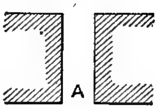


FIG. 85.

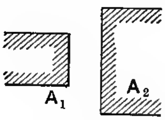


FIG. 86.

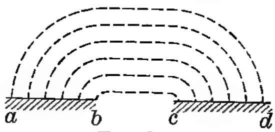


FIG. 87.

measured in square centimetres. In Fig. 85 this is simply $\frac{l}{A}$, in Fig. 86 it is $\frac{l}{\frac{A_1 + A_2}{2}}$. In more complex cases, such as in

Fig. 87, *ab*, and *cd*, the approximate path of the flux may be sketched to scale and the average length and area of the path estimated.

29. **Magnetic Hysteresis.**—*a. Hysteresis.*—If a ring of iron is magnetized by a steadily increasing force, and this force is then steadily diminished, it is found that the *B-H* curves in the two cases do not coincide. The *B-H* curve taken while the magnetizing force is decreasing lies entirely above the similar curve traced for an increasing magnetizing force. (See Fig. 88, curves *a* and *b*.) When the magnetizing force is removed the induction does not fall to zero. In other words, the changes in induction do not correspond to the changes in magnetizing force, but lag behind them. This *lagging* of the magnetism has been given the name **hysteresis.** If after bringing the force *H* to zero it is again steadily increased, the resulting *B-H* curve does not coincide with either of the others, and again the lagging change of magnetism is noted to change as the curve *c*, Fig. 88. In Fig. 89 are shown the relations between *B* and *H*, when *H* is alternately increased and diminished. It is to be noted that these curves form closed loops. When the ring of iron is magnetized by an alternating current the magnetizing force is not only increased and decreased but is reversed in direction. The cyclic relation between *B* and *H* for a bar magnetized in this manner is shown in Fig. 90. Three separate curves are there shown, giving this relation for three different alternating values of *H*. Each of these is a closed loop.

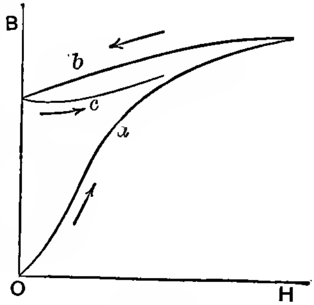


Fig. 88.

b. Energy Expended in Overcoming Hysteresis.—It has

been shown that energy exists in a magnetic field. A field set up while a current is increasing contains stored energy, which helps to maintain the current when it decreases. If the

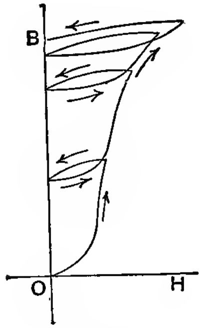


FIG. 89.

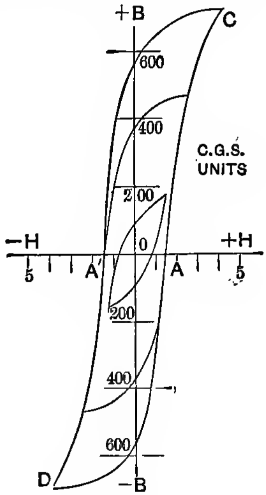


FIG. 90.

field is entirely in air it completely disappears with the current. In this case precisely the same amount of electrical energy is given back to the circuit by the destruction of the field as was previously required from the circuit to establish it. If the field is wholly or partly in iron or other magnetic material, it does not completely disappear when the current that established it ceases. (See Fig. 88.) The energy required to establish the field is not entirely restored to the circuit. To demagnetize the iron completely requires the expenditure of energy in the opposite sense, or a magnetizing force, O*A′*, Fig. 90, must be applied. If the then demagnetized iron be magnetized oppositely and demagnetized, the curve, *A′DA*, is traced and additional electric energy is expended. It is obvious, then, that to carry the iron through a complete magnetic cycle, tracing one of the loops in Fig. 90, *a definite quantity of*

electrical energy must be expended in overcoming magnetic hysteresis.

c. Derivation of Expression for Energy Dissipated in Hysteresis.—Suppose a bar of cross-section A, and length l, to be magnetized by a coil of n turns uniformly wound upon it. Let the length of the bar and coil be great in comparison with their diameters, so that H may be uniform throughout. Let the current increase slightly, causing a corresponding increase in B. The total flux in the bar is increased an amount $A \cdot dB$. This induces in the coil a counter-e.m.f., $nA\frac{dB}{dt}$. If i be the value of the current during this time, the work done is

$$dW = nAi\frac{dB}{dt}dt = nAidB.$$

Since the volume of the bar is lA, the work done per *cu. cm.* is $\frac{nidB}{l}$. The magnetizing force, H, per *cm.*, is $\frac{4\pi in}{l}$, when i is in *c.g.s.* units; substituting the work becomes

$$dW = \frac{1}{4\pi}HdB.$$

$$W = \frac{1}{4\pi}\int HdB = ergs, \quad \ldots \quad (84)$$

as the energy expended per *cu. cm.* per cycle of magnetization.

The power expended in watts per *cu. cm.* is

$$\frac{1}{4\pi T \cdot 10^7}\int HdB, \quad \ldots \quad (85)$$

where T is the time in seconds occupied by one complete cycle of magnetization.

The value of $\int HdB$ taken around one complete loop is the area of the loop. This area is, therefore, proportional to

the energy dissipated in carrying the iron through a complete cycle of magnetization.

d. Effect of Physical Treatment upon Hysteresis.—In Fig. 91 are shown two hysteresis loops from the same sample of

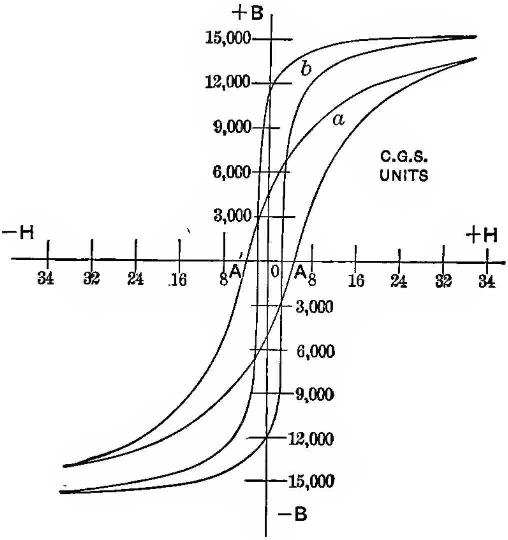


FIG. 91.

soft iron wire. Curve *a* is for the sample after being hardened by *stretching*, and curve *b* after thorough *annealing*. Note that the area of the former loop is greater. This shows that the work done in carrying the iron through a complete magnetic cycle is greater for the hardened wire, as the magnetizing force H varies between the same limits in each case. Note also that the flux density at similar values of H is greater in the annealed iron. These facts actually go together.

Hysteresis losses are affected by temperature. For all ordinary temperatures up to 200° C. but little change takes place. At higher temperatures these losses grow less, reaching from one fifth to one tenth of their original values in the

neighborhood of 700° C. Above 800° C. irons lose their magnetic properties. These temperature-hysteresis properties have served no useful purpose in engineering.

In most irons and electrical steels the hysteresis quality deteriorates with age, i.e., the hysteresis losses increase slowly with time. This process is called **aging.** Whenever electrical steel sheets are used in alternating current machinery the highest practicable quality in regard to hysteresis is sought. As these losses increase with age in different degrees for different treatments applied in the manufacture of the sheets much attention has been given to the conditions under which sheets may be produced that display an average minimum hysteresis loss. It has been found that the chief factor in the treatment of the steel sheets to secure minimum aging consists in annealing the sheets from the highest practicable temperature. This temperature is about 900° C. Above that temperature the sheets tend to stick together and form excessive scale. Solid irons and steels might be annealed at higher temperatures, a useless advantage in this respect on account of the fact that magnetic circuits for alternating flux must always be built up out of sheets to avoid eddy current losses.

The aging process further depends upon the temperature of the sheets in the electrical machinery under the conditions of actual use. It has been found through extensive tests that high permeability electrical steel sheets annealed from the highest temperature do not age much when not subjected to a higher operating temperature in the machinery or apparatus than 60° C., while serious aging will result if the operating temperature is raised to 90° C.

The following table of results given by Parshall and Hobart from actual tests show the effects of annealing and aging on the hysteresis quality of a particular sample of steel sheets:

Time in Hours.	Temperature Deg. C.	Hysteresis-loss in Watts per lb. at 100 Cycles per Second and at a Density of 24,000 Maxwells per *sq. in.*	
		Unannealed.	Annealed.
0	60°	0.382	0.325
600		0.382	0.325
1000	90°	0.390	0.405
1500		0.400	0.415

e. Effect of Impurities on Hysteresis.—Experimental investigations have not yet establised definite relations of impurities in irons and steels to their corresponding hysteresis properties. It is understood clearly, however, that the effects of physical treatment are far more important than those traceable directly to the presence of impurities.

In general, when the impurities are so proportioned as to produce the softest and most magnetically permeable iron or steel, the minimum hysteresis losses result.

The presence of certain impurities in steel, chief among which is combined carbon, together with hardening, mechanical and temperature variation treatment, will enormously augment the hysteresis and, therefore, the *permanent* m.m.f., a measure of which is OA or OA', Fig. 91. The larger this permanent m.m.f. the better will the specimen be suited for the construction of **permanent magnets.**

f. Comparison of Hysteresis Curves.—In Fig. 92 are two curves taken from modern brands of electrical steel sheets. The abscissæ show the energy expended, in ergs per *cu. cm.*, in carrying the steel through a complete magnetic cycle, the maximum flux densities being plotted as ordinates. The greater part of this energy is used in overcoming hysteresis. Other losses, however, mainly dependent on the thickness of the sheet, were present. These will be discussed later.

Different brands of iron and steel, under similar conditions, show great differences in the energy required for overcoming

hysteresis. In recent years great attention has been paid to the production of electrical steels showing small hysteresis losses and great permeability. The left-hand curve, Fig. 92, shows the characteristics of one of the best commercial grades. The right-hand curve, Fig. 92, is from a sample not quite so

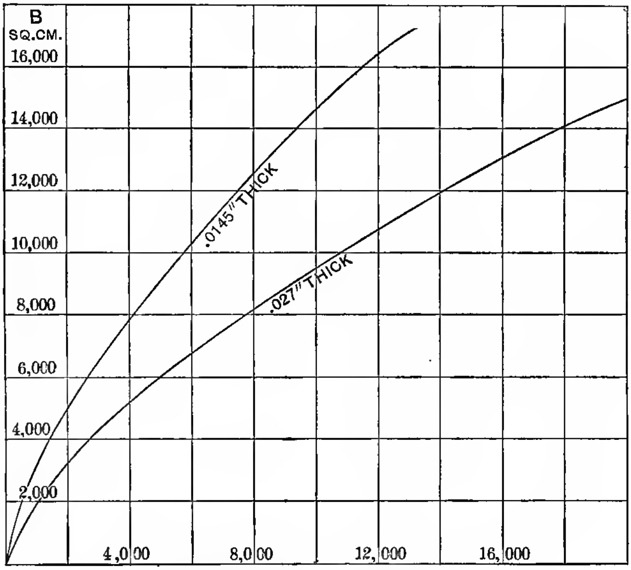


FIG. 92.—Hysteresis and Eddy Losses in Sheet Steel. Determined at 100 p.p.s.

good, but still to be regarded as excellent. Part of the greater loss shown for this sample is due to eddies on account of its greater thickness, and is independent of its magnetic quality.

The hysteresis losses per cycle in irons and steels occur as shown in given amounts dependent upon the maximum flux density per cycle per unit volume and independent of the time occupied per cycle and therefore of the frequency of the magnetic flux.

Steinmetz has shown that the relations between hysteresis losses and magnetic flux densities in irons and steels obey the law*:

$$W_h = \eta B^{1.6}, \qquad (86)$$

* Steinmetz, *Elec. Eng.*, Vol. X, p. 677.

where η is the quality-constant determined by the system of units employed, and the hysteresis character of the iron or steel. Using maxwells per sq. in. and ergs per cycle per *cu. in.*, the following values of η are found to occur in

The highest quality...................... .001
Excellent commercial qualities............ .0015
Poor qualities.......................... .002 and higher.

The following example illustrates the use of Steinmetz's law:

Required the watts lost through hysteresis in 1000 *cu. in.* of best quality commercial electrical steel sheets operated at a maximum alternating induction of 30,000 maxwells per square inch and at 100 cycles per second.

Solution:

$$\text{Watts lost} = \frac{.0015 \times 30{,}000^{1.6} \times 1000 \times 100}{10_7} = 216. \quad \textit{Ans.}$$

30. Ewing's Theory of Magnetism.—Weber suggested that the individual molecules of a magnetic substance are themselves magnets and that these magnets are more or less free to turn about, under the influence of outside forces. This theory evidently accounts for such a substance becoming a magnet under the influence of such forces. Weber, Maxwell, and others have suggested theories, based on this one as fundamental, to account for hysteresis, the *B-H* relation, etc. These theories involve various directing forces, frictional resistance, effects of stress, etc., which complicate matters. Ewing has simplified the whole matter by assuming nothing more than the original idea of small magnets, free to turn. He has shown, both theoretically and by means of working models, that the mutual action of the molecular magnets upon each other is quite sufficient to account for most of the observed phenomena.

The theory is as follows: The small magnets, when not under the influence of outside forces, form stable groupings. (See Fig. 93*a*.) When a small magnetizing force is applied

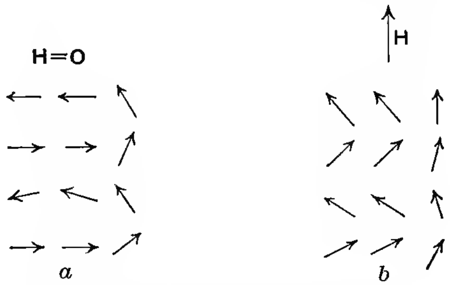


FIG. 93.—Ewing's Magnet Model.

to a group the individual molecules turn slightly in the direction of that force. (See Fig. 93*b*.) If the force is removed, the old groupings are again formed. This corresponds to the lower part of the B-H curve.

If a stronger force be applied, a point is reached where the old groups begin to break up, the magnets swinging violently around, forming new groupings which have a general aligning tendency in the direction of the applied force, though no individual magnet may be exactly in that direction. This condition of groups, just on the point of breaking up, corresponds to the second stage of magnetization indicated by the steepest part of the typical B-H curve, where a small increase in H causes a large change in the induction. If the outside force is removed during this stage or later, the magnets do not return to their original groupings, but the bar remains more or less strongly magnetized. (See curve *b*, Fig. 88, and Fig. 94*c*.)

If, after the original groups have been thoroughly broken up, the magnetizing force is still further increased, the newly formed groups are not broken up, for the magnets composing

them are already turned more or less into the direction of the force. The magnets, however, swing more and more nearly into line as the force is indefinitely increased, an infinite force being theoretically required to make them exactly parallel with each other. This corresponds to the upper part of the *B-H*

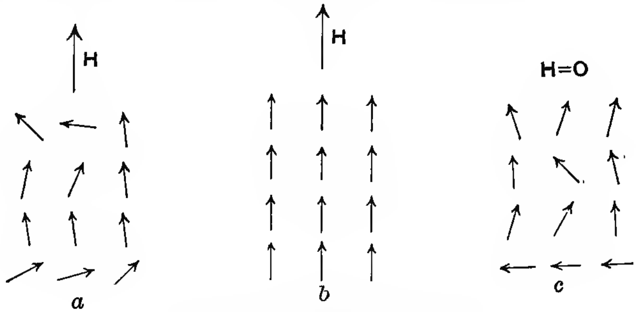


FIG. 94.—Ewing's Magnet Model.

curve, or to the saturation of the iron. If the magnetizing force is gradually removed during this stage, the *B-H* curve traced on the return very nearly coincides for a time with that drawn for an increasing *H*. (See upper part of curve *b*, Fig. 91.)

The magnet groupings in Figs. 93*a*, 93*b*, 94*a*, 94*b*, and 94*c* have been formed by an experimental model containing twelve little magnets .25 inch long and pivoted upon needle points .45 inch apart. Fig. 93*a* was traced from the model when subjected to no m.m.f., the earth's field having been neutralized. Fig. 93*b* was traced from the model after a small m.m.f. had been applied by bringing into the neighborhood a bar magnet. This state of the model corresponds to the earliest portion of the *B-H* curve in irons and steels. Upon removing the bar the magnets regained their original positions given in Fig. 93*a*. The bar magnet was then brought nearer so as to apply a stronger m.m.f. when the magnets arranged themselves as shown in Fig. 94*a*. The positions of the little

magnets represent now the average molecular condition in iron for the steepest portion of the *B-H* curve. On bringing the bar still nearer, all of the little magnets arranged themselves in the direction of the externally applied m.m.f. (see Fig. 94*b*), corresponding to the condition of saturation in magnetized soft iron. The bar was then removed entirely from the neighborhood of the model and the little magnets grouped themselves as shown in Fig. 94*c*, which corresponds to the lower end of the curve *b* in Fig. 88.

Thus it is seen that if a magnetic body like iron is made up of molecules which are actual magnets free to swing through any angle in their own positions, and in this swinging dissipate energy, all the essential phenomena of magnetic induction in iron are accounted for by Ewing's theory. When most of the molecular magnets are arranged approximately in a single direction, the entire mass of iron becomes a powerful magnet, establishing through itself a large flux due to the influence of a comparatively small actuating or impressed m.m.f. externally applied.

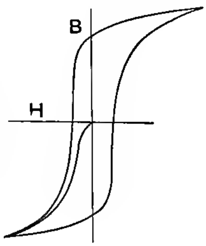


FIG. 95.
Hysteresis Card obtained by Ewing from Magnet Model.

Professor Ewing's working model contained twenty-four pivoted magnets. By means of a delicate instrument he measured the *B-H* relation for the flux set up through this model and a small actuating m.m.f. externally applied in a cycle of gradual change, and obtained thereby the hysteresis card given in Fig. 95.

31. Illustrative Problems.—Prob. 49. Required the reluctance of the magnetic circuit shown in Fig. 79 when the total induction is 290,000 maxwells. The dimensions and materials are as follows:

Bar *NS*, 16″ by 3″ by 3″, cast steel.

Bar AAA, 30″ by 3″ by 3″, cast iron.

Air-gaps, $\frac{1}{8}''$ each.

Solution: The reluctance of each part of the circuit is given by the formula

$$\mathcal{R} = \frac{l}{a\mu},$$

and the permeability is obtained from the B-H curves of Fig. 80, by reducing the values of B and H to the *c.g.s.* system and then substituting in the expression

$$\mu = \frac{B}{H}.$$

After reducing the dimensions to centimeters and square centimeters the reluctances are as follows:

$$\mathcal{R}_s = \frac{40.6}{918 \times 58} = .000762;$$

$$\mathcal{R}_i = \frac{76.2}{210 \times 58} = .006256;$$

$$\mathcal{R}_{ag} = \frac{.635}{58} = .010948.$$

The total reluctance is the sum of these, or .017966. *Ans.*

Prob. 50. How many ampere-turns must be applied to the magnetic circuit of Prob. 49 in order that the required induction may be set up?

Solution:

$$\text{M.m.f.} = .017966 \times 290{,}000 = 5210 \text{ gilberts.}$$

The corresponding ampere-turns are

$$It = \frac{\text{M.m.f.}}{.4\pi} = \frac{5210}{1.257} = 4145 \text{ ampere-turns.} \quad \textit{Ans.}$$

Prob. 51. How many ampere-turns must be applied in order to set up twice this induction?

Solution: 10,000 maxwells per sq. cm. correspond to 64,500 per sq. inch. From the curves 15 ampere-turns per inch are required in the steel and 290 in the iron. Total ampere-turns in steel are

$$16 \times 15 = 240.$$

Total ampere-turns in iron are

$$30 \times 290 = 8700.$$

In the air-gaps the m.m.f. in gilberts is

$$\text{m.m.f.} = .01094 \times 580{,}000 = 6345,$$

which reduces to

$$\frac{6345}{1.257} = 5047 \text{ ampere-turns.}$$

The total m.m.f. then becomes

$$240 + 8700 + 5047 = 13{,}987 \text{ ampere-turns.} \quad \textit{Ans.}$$

The cast iron consumes the largest part of this.

Prob. 52. What is the total pull in pounds as measured by the apparatus shown in Fig. 79, when B is 5000 maxwells per sq. cm.?

Solution:

$$\text{Pull in dynes} = \frac{AB^2}{8\pi} = 57{,}600{,}000.$$

$$\text{Pull in pounds} = \frac{57{,}600{,}000}{445{,}000} = 129.4 \text{ lbs.} \quad \textit{Ans.}$$

Prob. 53. In Fig. 84 the "leakage coefficient" is 1.4. This means that 1.4 times the induction desired in the armature must be produced, the remainder forming the leakage field. How many ampere-turns must be wound on the vertical cores,

AA, in order that an armature induction of 7,000,000 maxwells may be produced?

The constants of the circuits are as follows:

Horizontal yoke, *A*, 11″ by 11″ by 28″.

Vertical cylindrical cores, *AA*, diameter 10.5″, length 16″.

Pole-pieces, height 12″, width perpendicular to paper 12″.

Armature core, diameter 10″, length 11″.

Air-gap, double length 1.18 in., approximate area 152 sq. in.

Solution: The approximate length and area of the path of the flux must be found for each part of the circuit. This is as follows:

Yoke, length 28 in., area 121 sq. in.

Two cores, length 32 in., area 86.6 sq. in.

Two pole-pieces, length 10 in., area 144 sq. in.

Two air-gaps, length 1.18 in., area 152 sq. in.

Armature (actual iron), length 11.8 in., area 80 sq. in.

The total induction of 9,800,000 maxwells (7,000,000 × 1.4) is assumed to pass through the yoke when 7,000,000 maxwells are set up in the air-gaps and armature core. That in the cores may be estimated at 9,000,000 maxwells and the pole-pieces will receive about 8,000,000 maxwells. The induction densities are as follows:

B in yoke	=	81,000	maxwells	per	sq. in.
B in cores	=	104,000	"	"	"
B in poles	=	55,500	"	"	"
B in air-gaps	=	46,000	"	"	"
B in armature core	=	87,500	"	"	"

The corresponding values of ampere-turns as determined by multiplying the m.m.f. per inch (from Fig. 80) by the length of each part are:

In cast-steel yoke, 25×28	$=$	700 ampere-turns.	
In cast-steel cores, 115×32	$=$	3,680	"
In cast-steel poles, 12×10	$=$	120	"
In sheet-steel armature core, 30×11.8	$=$	354	"
In air-gap, $\dfrac{46{,}000}{6.45} \times \dfrac{1.18 \times 2.54}{.4 \times \pi}$	$=$	17,000	"
Total ampere-turns	$=$	21,854.	*Ans.*

Prob. 54. The permeability of a certain specimen of iron is 1000 at an induction density of 12,000 maxwells per square centimetre. How much m.m.f., in gilberts, is required to maintain this density in a rod of this iron one foot in length?

365.75 gilberts. *Ans.*

Prob. 55. A cast-steel ring, one foot in mean diameter and of one square inch cross-section, is subjected to a magnetizing force of 1000 ampere-turns. By means of the curves shown in Fig. 80* find the total induction produced.

81,000 maxwells. *Ans.*

√ Prob. 56. Calculate the reluctance of the cast-steel and air magnetic circuit represented in Fig. 96, when the induction density is 100,000 maxwells per square inch. Determine also

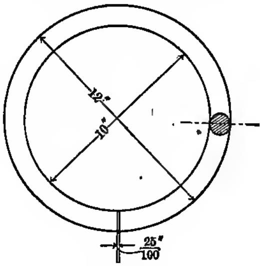


FIG. 96.

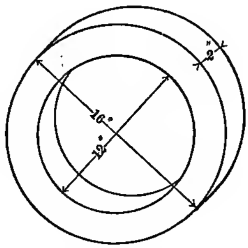


FIG. 97.

the m.m.f. in ampere-turns necessary to maintain this density.

Reluctance .17089. *Ans.*

10,678 ampere-turns. "

* These curves will be used in subsequent problems in this chapter.

Prob. 57. 8000 ampere-turns surround the magnetic circuit shown in Fig. 97. The ring is of cast iron. How much flux will be established in the ring?

220,800 maxwells. *Ans.*

Prob. 58. The transformer core shown in Fig. 98 is made of electrical steel laminations. How many ampere-turns must

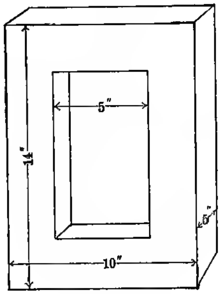


FIG. 98.

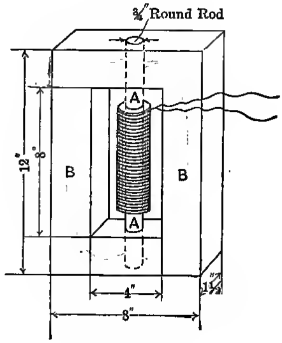


FIG. 99.

be applied to this magnetic circuit to set up an induction of 500,000 maxwells?

570 ampere-turns. *Ans.*

Prob. 59. The apparatus shown in Fig. 99 is used for determining the magnetic properties of materials. The rod *AA*, which is being tested, is of cast iron and the yoke *BB* is of cast steel. Determine the density of induction per square inch which will exist in *AA* and in *BB* when a m.m.f. of 1000 ampere-turns is supplied by the coil.

Neglect the reluctance of that part of rod *AA* when it passes through the yoke *BB*. Note that the magnetic circuit in the yoke is divided into two paths.

Method: Assume several values of the induction and plot a curve between induction and total ampere-turns. From this the induction corresponding to the given number of ampere-turns can be readily determined.

In *AA* approximately 48,500 maxwells per square inch. *Ans.*
In *BB* " 3,570 " " " " "

Prob. 60. Fig. 100 shows a magnetic circuit of cast steel consisting of four parts: a U-shaped piece, two air-gaps, and a keeper or armature. With a m.m.f. of 3000 gilberts applied

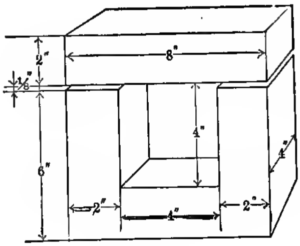


FIG. 100.

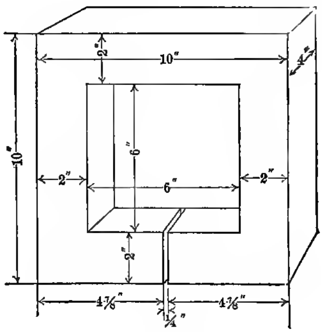


FIG. 101.

find the total induction produced. How many ampere-turns correspond to 3000 gilberts? 226,000 maxwells. *Ans.*

2390 ampere-turns. "

Prob. 61. It is desired to produce an induction of 1,000,000 maxwells in the sheet-steel magnetic circuit shown in Fig. 101. How many ampere-turns are necessary to do this?

25,020 ampere-turns. *Ans.*

CHAPTER VIII.

ROTATING MAGNETIC FIELDS.

SYNOPSIS.

32. Polyphase e.m.f.s, currents, and fields.
    - *a.* Polyphase quantities.
    - *b.* The rotating magnetic field.
        1. Pivot fields.
        2. Cylinder fields.

33. Components of the rotating pivot fields.
34. Production of a rotating pivot field.
    - *a.* By means of two-phase currents.
    - *b.* By means of three-phase currents.
    - *c.* Irregular rotating pivot fields.
    - *d.* Limited use of pivot fields.

35. Components of the rotating cylinder magnetic field.
36. Practical production of rotating cylinder magnetic fields.
    - *a.* Limited number of phases employed.
    - *b.* The cylinder fields formed by two- and three-phase currents fluctuate.
    - *c.* Rotating cylinder fields produced by two-phase currents.
    - *d.* Rotating cylinder fields produced by three-phase currents.

32. Polyphase E.m.f.s, Currents, and Fields.—*a.* In modern electrical engineering frequent use is made of two, three, four, or six and sometimes more associated alternating pressures, currents, and magnetic fields in electrical machinery and its connecting circuits and auxiliary apparatus. This multiple arrangement of alternating quantities is designated as **polyphase.** Polyphase is the general term used to designate that more than one alternating quantity is in use. The corresponding terms for actual cases are **two-phase, three-phase, four-phase,** etc. Polyphase e.m.f.s, currents, and magnetic fields are equal in value and differ equally in phase.

In machinery the coils forming the several circuits carrying the several currents and the magnetic fields they establish are spaced equally with respect to one another.

Polyphase magnetic fields give rise to uniform rotating magnetic fields, and are very useful for a variety of purposes in engineering. Their significance will be better understood by means of the following illustrations that rotating magnetic fields and their components afford.

b. The Rotating Magnetic Field.—Rotating magnetic fields as applied in engineering occur in two classes, which may be called pivot and cylinder fields.

1. *A pivot field* may be any single field established in air or other mobile medium wherein the field is made to revolve about a definite axis located approximately at the centre of the field and at right angles to the flux.

In Fig. 102 the circular coil *CC*, carrying a continuous current, establishes a field of magnetic flux distributed as the lines in the figure show. When the coil is stationary and the current constant the field of flux is correspondingly stationary and constant. If the coil be rotated about an axis passing through its own plane and centre of the field of flux, so long as the current is maintained in the coil the flux will rotate likewise precisely as though it were a rigid body attached to the coil. Such a field, in whatever manner produced, for the sake of clearness will be called a **pivot field.**

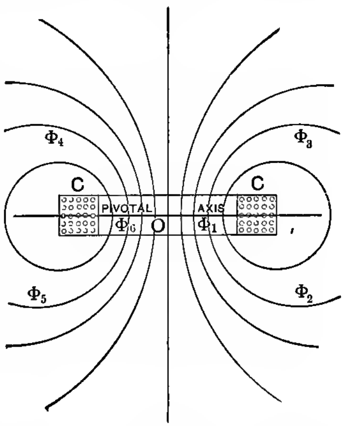


FIG. 102. — Model of Rotating Pivot Field.

2. *A cylinder field* consists of cöordinated sets of positive

and of negative fields of flux established through a cylindrical air-gap rotated about the axis of the air-gap cylinder.

Fig. 103 illustrates the production of such a field in the

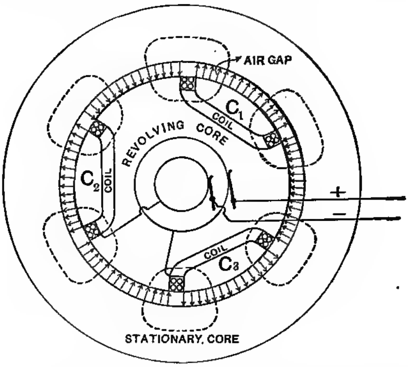


FIG. 103.—Model of Rotating Cylinder Field.

simplest possible manner. This model employs two magnetic cores built of electrical sheet steel. The outer core forms a cylindrical shell; it is stationary and is mounted concentric with an inner cylinder built up in the same manner. The inner core rotates. Upon it in notches or grooves are mounted coils carrying continuous current introduced through slip-rings as indicated. Between the inner and outer cylinders there is a clearance space or air-gap. The coils while carrying current establish magnetic fields through and about themselves, as indicated in Fig. 103. When the core upon which these coils are mounted rotates, the coils and the fields of flux they produce rotate with them.

These rotating cylinder fields are used extensively in modern electrical machinery where they are invariably produced by means of alternating currents, as will be shown later. The particular model above used has but little application. It has been introduced here for the express purpose of making clear precisely what is meant by **rotating cylinder field.**

33. Components of the Rotating Pivot Field. — Any uniform stationary single field of flux, such as illustrated in Fig. 102, may be broken up into components along any set of axes. When such field rotates its components remain stationary in position and alternate in value. If the field rotates uniformly, it will be seen that the components alternate as sine values.

Algebraically, it is more convenient to study components taken at right angles than along axes at other angles. A rotating field may, however, be resolved into components along any axis by writing each component in terms of its horizontal and vertical components.

Graphically, one set of axes are as convenient as another.

b. Rectangular Components of a Pivot Field.—In Fig. 104 let Φ be a uniform field of magnetic flux parallel to the plane

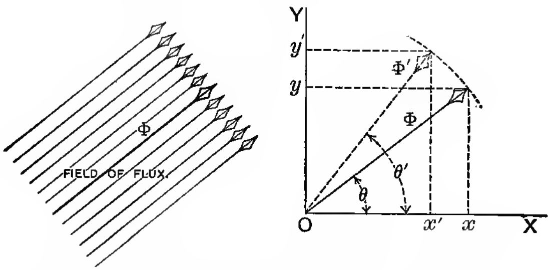


FIG. 104.—Rectangular Components of the Pivot Field.

of this paper and making the angle θ with the horizontal or X axis. It may be assumed that this uniform field of flux was established over a limited space at the centre of the coil CC in Fig. 102. The field at that point is quite uniform for small distances in any direction from the centre of the coil.

The horizontal component of this field is

$$\Phi \cos \theta \quad \text{or} \quad \Phi \sin \left(\theta + \frac{\pi}{2}\right),$$

and the vertical component is

$$\Phi \sin \theta.$$

Suppose the field Φ be rotated so as to make a new angle θ' with the horizontal. The horizontal and vertical components will now be

$$\Phi \sin \left(\theta' + \frac{\pi}{2}\right),$$

$$\Phi \sin \theta'.$$

It is evident that a continuation of the angular displacement of the flux, Φ, so as to pass through a complete revolution will at the same time cause the horizontal and vertical components to vary as curves of sines throughout one complete cycle.

When the field rotates uniformly, therefore, the horizontal and vertical components of such field constitute simple fixed alternating fields of flux that are equal and in quadrature phase. From this it follows that

Two equal simple alternating fields of flux located at right angles, displaced in phase by one quarter of a cycle, will produce a uniform rotating magnetic field.

34. Production of a Rotating Pivot Field.—*a. By means of Two-phase Currents.*—In Fig. 102 is shown a coil carrying current which sets up a field of flux that in density and direction is given by the curving lines. When the current alternates, the field of flux alternates in the same manner but without change of relative density and direction, i.e., the character of flux distribution remains the same. At the centre of this coil there is a small region in which the field is uniform in amount and direction. Keeping these facts in mind, the field set up at the centre of the coils in Fig. 105 may be studied.

These coils 1 and 2 are circular in form and mounted at right angles to one another with their centres at a common point. An alternating current is set up through coil 1. In

coil 2 there is set up an alternating current equal in strength to and in quadrature with that in 1.

These coils will establish at their centres alternating flux densities given in maximum amount, direction, and phase by the vectors B_1 and B_2 respectively. If the currents vary as sines, the horizontal and vertical flux densities B_1 and B_2 will vary as sines and be exactly equal to the horizontal and ver-

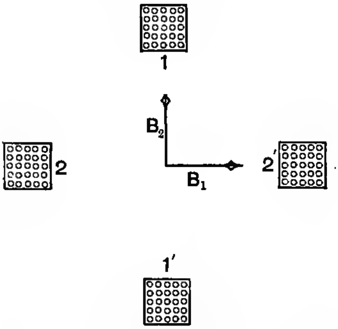


FIG. 105.—Production of a Pivot Field by Means of Two-phase Currents.

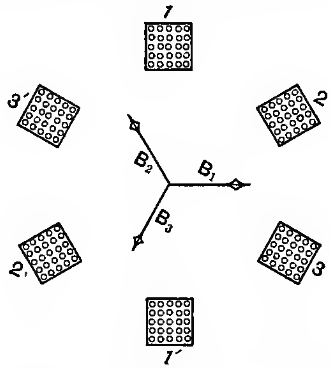


FIG. 106. — Production of a Pivot Field by Means of Three-phase Currents.

tical components of a revolving field. Their combination will, therefore, produce a revolving field. The flux density value of the revolving field will be

$$B = \sqrt{\overline{B_1 \sin \theta}^2 + \overline{B_2 \sin \left(\theta + \frac{\pi}{2}\right)}^2}.$$

Since B_1 and B_2 are equal, on substituting any value of θ we have

$$\textit{Revolving flux density } B = B_1 \text{ or } B_2.$$

Equal sine wave currents in quadrature, passed through coils mounted at right angles, set up at their common centre a rotating magnetic flux that is constant in density and angular velocity.

b. By Means of Three-phase Currents.—In Fig. 106 three coils, Nos. 1, 2, and 3, are mounted about a common centre and spaced with symmetry 120° apart. Assume that an alternating current passes through each of these coils; that such currents are equal and that they differ in phase from each other by 120°. Each coil will establish a uniformly distributed alternating field of flux in a small region about its centre. The direction of this flux will be at right angles to the plane of the coil. The three coils will establish three components of a rotating flux that is uniform in density and angular velocity. This follows from the fact that when any uniform field rotating at uniform velocity is broken up into components along axes taken at 120°, such components constitute an arrangement of three-phase alternating fluxes identical as to character with three fluxes formed at the centres of the coils in Fig. 106. Some particular value of constant, uniformly rotating field when broken up in this way will resolve into components identical in character and amount with the field formed at the centre of the coils. *The combination of these fields must produce that same value of uniformly rotating field.*

The value of the rotating field formed by a three-phase field is easily determined, since it is only necessary to add the three fields at any instant. This can be accomplished by resolving the fields into rectangular components. The resultant field will then be obtained at any instant by taking the square root of the sum of the squares of the sums of vertical and horizontal components respectively.

Thus the values of the three-phase fields as functions of time are

$$B_1 \sin \theta,$$
$$B_2 \sin (\theta + 120°),$$
$$B_3 \sin (\theta + 240°),$$

where θ varies uniformly with time.

Coil No. 1 is located in the vertical axis; it will then establish flux in the horizontal axis which does not have to be resolved into components. The coils 2 and 3 will establish flux densities B_2 and B_3, 120° and 240° remote from B_1, and as shown in Fig. 106.

The values of the vertical components of B_2 and B_3 will therefore be

$$\sin 120^\circ\, B_2 \sin(\theta + 120^\circ) = + .866 B_2 \sin(\theta + 120^\circ);$$
$$\sin 240^\circ\, B_3 \sin(\theta + 240^\circ) = - .866 B_3 \sin(\theta + 240^\circ);$$

and the horizontal components will be

$$\cos 120^\circ\, B_2 \sin(\theta + 120^\circ) = - \tfrac{1}{2} B_2 \sin(\theta + 120^\circ);$$
$$\cos 240^\circ\, B_3 \sin(\theta + 240^\circ) = - \tfrac{1}{2} B_3 \sin(\theta + 240^\circ).$$

The sum of the vertical components will be

$$.866 B_2 \sin(\theta + 120^\circ) - .866 B_3 \sin(\theta + 240^\circ),$$

and of the horizontal components

$$B_1 \sin\theta - \tfrac{1}{2} B_2 \sin(\theta + 120^\circ) - \tfrac{1}{2} B_3 \sin(\theta + 240^\circ).$$

The value of the rotating field will be the combination of these horizontal and vertical components.

$$\textit{Rotating } B = \{.866^2[B_2 \sin(\theta + 120^\circ) - B_3 \sin(\theta + 240^\circ)]^2$$
$$+ [B_1 \sin\theta - \tfrac{1}{2}\{B_2 \sin(\theta + 120^\circ) + B_3 \sin(\theta + 240^\circ)\}]^2\}^{\frac{1}{2}}.$$

On substituting any particular value of θ and denoting $B_1 = B_2 = B_3$ by $B_{\text{max.}}$ we have

$$\textit{Rotating } B = 1.5 B_{\text{max.}}.$$

c. Irregular Rotating Pivot Fields.—Practical application of the pivot field is made occasionally where the components forming the same differ in amount and phase position. The above treatment applies to these fields also when note is made of the fact that the values of B_1, B_2, etc., are different in

amount and that they have their own different phase positions, θ_1, θ_2, etc.

Irregular components produce rotating fields that fluctuate in value and angular velocity.

d. Limited Use of Pivot Fields.—But a limited analysis needs to be made of the pivot field in free air or a homogeneous medium. There is little need in engineering for determining values of rotating fields established completely in the air. Such fields are only used in instruments and for demonstrational experiments.

The rotating cylinder fields are far more useful in engineering on account of their application in the induction motor, frequency converters, phase transformers, etc., where they are formed by multiple fixed cylindrical alternating fields.

35. Components of the Rotating Cylinder Magnetic Field.—In Fig. 103 assume that the coils are spread out and mounted in a number of small grooves instead of being bunched each in a single set of grooves. Assume further that these coils have been spread out in such a fashion as to cause them to establish fields of flux which will have a circumferential density distribution through the cylindrical air-gap corresponding to the curve of sines. In the air-gap over the surface of the magnetic cores the flux densities will rise and fall in positive and negative values as sine waves. After assuming also that the inner core, coils, and field are revolving at a uniform angular velocity there will exist in the air-gap cylinder of this model a rotating cylinder field having the form applied in engineering and for which the components are to be determined.

It is important that a clear idea be gained of the character of the rotating field here specified. To assist in accomplishing this the cylinder field has been developed in Fig. 107. The arrow-heads show by their length and position the flux density as it occurs from point to point in the field. Assume that the

direction of rotation is such that the waves move in Fig. 107 from left to right. Since this diagram is the development of a cylinder, the points A and B are identical. As a wave runs off at B it simultaneously appears at A. This cylinder field is here clearly seen to be one rotating at a constant velocity and having a circumferential sine-wave flux density distribution.

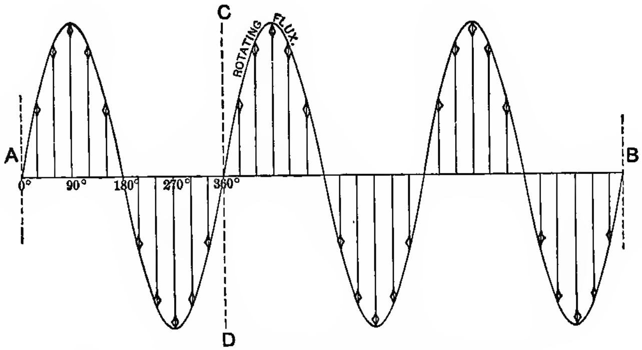


FIG. 107.—Development of Sine-form Cylinder Field.

The components of a cylinder field of this character will be best understood by noting the manner in which such a rotating field may be produced by means of a number of sets of coils carrying polyphase currents, and mounted symmetrically side by side on one of the cores in Fig. 103 in lieu of the coils there shown. To make this clear assume the actual case where nine circuits form nine sets of coils, each carrying one of nine polyphase currents. In Fig. 103 let each core be stationary. On the inner core the nine sets of coils are mounted symmetrically with respect to one another and the cylinder. Fig. 108 gives a diagram showing the manner in which these circuits or coils are mounted. $AA'BB'$ is a portion of the surface of the core developed whereon these circuits are mounted. As this diagram indicates, each circuit threads its way across the surface of the cylinder as many times as

there are to be poles or plus and minus regions of flux in the cylinder field.

The beginning and terminal ends of these circuits are numbered 1, 2, 3, 4, 5, 6, 7, 8, and 9, and 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, and 9′. In electrical machinery the portion of a circuit that crosses a core in each instance in the manner here applied is called an **inductor**. Assuming that there are to be

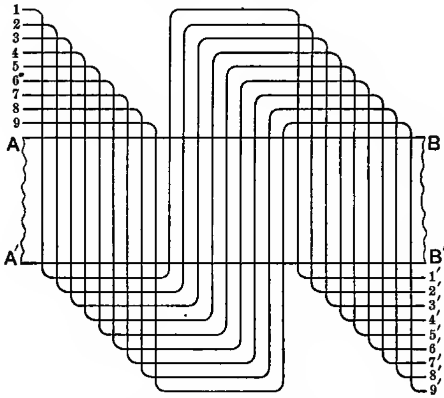


FIG. 108.—Shows the Arrangement of Nine Polyphase Circuits for the Production of a Cylinder Field.

six poles, i.e., three plus and three minus regions of flux, in this cylinder field, each circuit will cross the core six times, the terminal end being brought out at the same point on the core where the circuit begins. Thus each circuit contains *six inductors*. Since there are nine circuits there will be 9 × 6 or *fifty-four inductors, total.* These fifty-four inductors must cover the whole circumferential surface of the core symmetrically and as indicated in the diagram of Fig. 108.

In a model of this sort it is necessary to distinguish between the inductors according as the circuit passes through them from the front to the rear or *vice versa*. Those inductors through which the circuit passes from the rear to the front have a **positive position** and those through which the circuit passes

in the reverse order have a **negative position.** This distinction is made solely for the sake of convenience in distinguishing between the two directions in which the circuit is led across the core upon which it is mounted. It is also necessary to have a convenient means for referring to the positions occupied

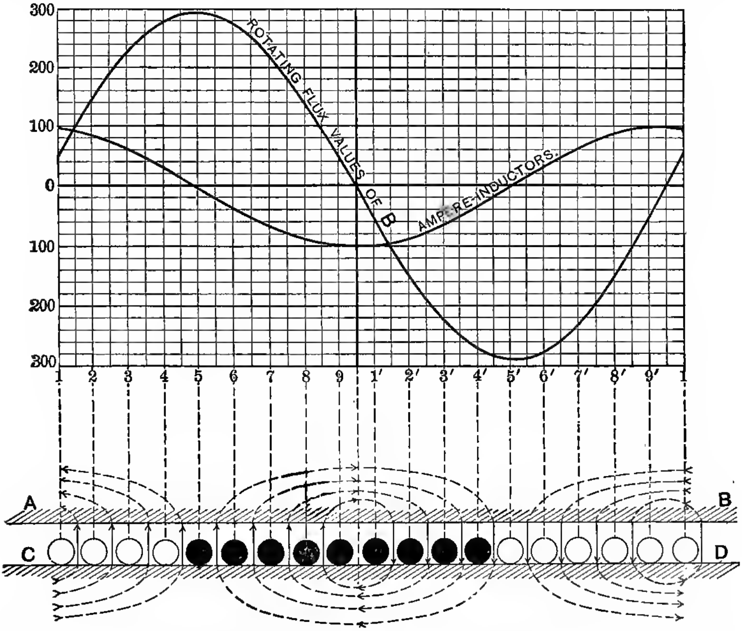


FIG. 109.—Formation of a Uniform Sine-wave Rotating Cylinder Field by Means of a Polyphase Arrangement of Circuits and Currents.

by the inductors on the surface of the core. To do this the circumferential distance on the surface of the core between neigboring inductors or sets of inductors of the same sign belonging to the same circuit is called a **polar interval.**

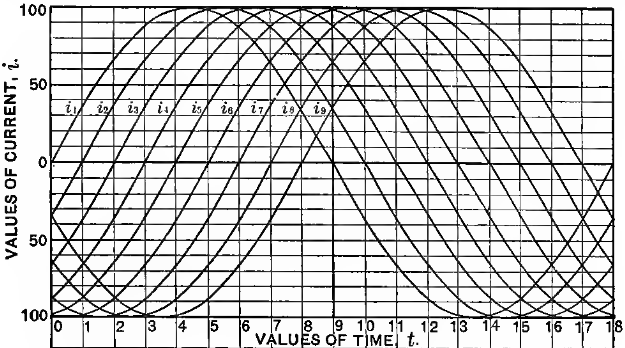
As stated above these fifty-four inductors occupy all of the surface of the core. A developed section for a single polar interval is given in the lower portion of Fig. 109. *AB* is taken along the surface of the outer core, and *CD* along the

surface of the inner core. The air-gap separating the two cores is AC or BD. It is seen that each inductor occupies a circumferential surface of *one eighteenth* of a polar interval or cycle of inductor-circuit connection. It is customary to refer to such a space in degrees, counting a polar interval 360°. The interval between inductor centres is, therefore, 20°. To form the rotating field a separate sine-wave alternating current must be passed through each of the nine circuits. These currents must be alike in periodicity and amounts, while they must differ in phase by an amount that corresponds to the circumferential displacement of their circuits. Neighboring currents must, therefore, differ in phase by 20°, corresponding to the position displacement of the circuits in which they exist.

A comprehensive diagram of this arrangement of inductors, circuits, and currents is given in Fig. 110. The nine sine waves of current-time $\dot{i}_1$, $\dot{i}_2$, $\dot{i}_3$, $\dot{i}_4$, $\dot{i}_5$, $\dot{i}_6$, $\dot{i}_7$, $\dot{i}_8$, $\dot{i}_9$, have been established through the corresponding circuits. If the length of one current wave in this diagram be allowed to represent also a polar interval, then the position at which the currents pass through zero may represent the location of the inductors of the corresponding circuits. Thus the numbers immediately under the heading *circuits* give the circuits to which the inductors in this part of the cylinder belong. To distinguish negative from positive inductors their corresponding inductors are marked prime.

From this diagram one may obtain the value of the current in any inductor and therefore in any circuit at any instant. It is possible to know from instant to instant the values and signs of all the currents in all the inductors, from which the resulting m.m.f. and therefore the fields they establish may be determined. This has been done for ten different instants, and the values of the currents and their direction across the core have been recorded in the table given in Fig. 110.

In order to note the m.m.f. of a circuit it is necessary that the value of the current and its direction in the circuit be known. It is necessary in this instance, therefore, to know whether



VALUES OF TIME, t.	CIRCUITS. 1	2	3	4	5	6	7	8	9	1′	2′	3′	4′	5′	6′	7′	8′	9′
0	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98	98	87	64	34
1	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98	98	87	64
2	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98	98	87
3	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98	98
4	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98
5	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87
6	87	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64
7	64	87	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34
8	34	64	87	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00
9	00	34	64	87	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**
10	**34**	00	34	64	87	98	98	87	64	34	**00**	**34**	**64**	**87**	**98**	**98**	**87**	**64**

VALUES OF CURRENT, i, IN EACH CIRCUIT

FIG. 110.—Shows the Formation of a Uniform Sine-wave Rotating Cylinder Field by Means of a Polyphase Arrangement of Circuits and Currents.

the current passes through the inductors from the rear to the front of the core or *vice versa*. It was stated above that inductors are positive when the circuit leads through them from the rear to the front. This same convention will be adopted for the sign giving the direction of a current in a circuit. A current has a positive direction in a circuit made up of induc-

tors when it will pass through a positive inductor from the rear to the front of the cylindrical core.

In the table positively directed currents occur in light-bodied type and those which are negatively directed occur in heavy-bodied type. A light value of current, therefore, passes through its circuit over the core from rear to front, while the black values do the opposite. The sign of the current direction will be positive when the signs of its inductor and itself are alike, and negative when not alike.

Let the instant $t = 0$ in the diagram be selected for the first determination of the m.m.f.s that are produced by these polyphase circuits. The vertical line at this instant will cut the nine current waves, and the distances from these intercepts to the axis of the waves measure the corresponding instantaneous currents, which at this moment are all negative. The current direction through the nine positive inductors at this instant is therefore minus and the values are recorded in black. The next series of nine inductors are negative, through which when negatively directed currents are passed a positive direction across the core results. The currents in these nine negative inductors have the same values as in their corresponding positive inductors, since they are in the same corresponding circuits. The same corresponding values of current are therefore always given for the negative inductors as for their positive companions except with opposite signs.

With this understanding the table can now be quickly made up for the other instants of time selected. The values of the currents at the instant $t = 0$ for any polar interval were:

1	2	3	4	5	6	7	8	9	1′	2′	3′	4′	5′	6′	7′	8′	9′
00	**34**	**64**	**87**	**98**	**98**	**87**	**64**	**34**	00	34	64	87	98	98	87	64	34

When $t = 1$ the value of the current in circuit No. 1 has increased from 0 to $+ 34$, while the currents in all of the other circuits have changed, remaining negative. The currents and

their directions in inductors of each polar interval have therefore become

1	2	3	4	5	6	7	8	9	1′	2′	3′	4′	5′	6′	7′	8′	9′
34	00	34	64	87	98	98	87	64	34	00	34	64	87	98	98	87	64

In precisely this same manner the currents and their directions may be determined for any other instant whatsoever. The table carries out the results for instants taken at convenient intervals up to $t = 10$ or for angles up to 200°. The table might have been made up for instants taken at any shorter or longer, regular or irregular interval. The results would have been the same as in this case, i.e., a regular forward movement of the positive and negative m.m.f.s.

The field of flux established by these moving m.m.f.s has been determined in connection with Fig. 109, corresponding to the instant $t = 4$. It is seen that inductors 5, 6, 7, 8, 9, 1′, 2′, 3′, and 4′ convey current across the core in a negative direction and are drawn in black section in the diagram. The remaining inductors in this polar interval, 1, 2, 3, 4, 5′, 6′, 7′, 8′, 9′, have current in them traversing the core in a positive direction and are drawn with clear section. The current-inductor status in the other polar intervals of this field is precisely the same as in the one given in this diagram. Over the circumference of the cylinder there are, therefore, alternate regions of nine positive and nine negative inductors. These present about themselves in closed circuits through the air-gaps and cores ampere-turns of m.m.f. as indicated by the closed curves in Fig. 109. It is seen that the maximum ampere-turns occur in closed circuits about the whole set of positive or negative inductors. The ampere-turns diminish in closed circuits within the space occupied by each positive or negative set until the minimum values of zero are reached at their centres.

The values of the currents in the inductors have been

plotted in rectangular coördinates in the upper portion of Fig. 109. This curve is a sine wave and is labelled *ampere-inductors*. Its maximum value is 100 ampere-turns, while the ampere-turns of inductors 9 and 1′ which lie on either side of the maximum have the values each of

$$100 \sin (4 \times 20^\circ) = 98.48,$$

or 98 as put down in the table.

The m.m.f. exerted about the inductors will be given by the sum of the inductor-currents included within each closed magnetic circuit taken symmetrically with respect to positive and negative sets.* These sums are given in the following table:

	Inductors. 5	6	7	8	9	1′	2′	3′	4′	5′	Total Ampere-turns.
Closed magnetic circuits	Amperes.										
through 5 and 5′..............	00	34	64	87	98	98	87	64	34	00	566
between 6 and 7 and 3′ and 4′..			64	87	98	98	87	64			498
" 7 and 8 and 2′ and 3′..				87	98	98	87				370
" 8 and 9 and 1′ and 2′..					98	98					196
" 9 and 1′ and 9 and 1′..					0	0					0

Compared with the reluctance of the air-gaps the reluctance of the iron cores may be neglected in this instance. The radial depth of each gap is .495 inch or 1.257 centimetres. Each flux path must cross the gap twice. The flux density at any point in the cylinder gap where a closed magnetic path crosses will be

$$B = \frac{\text{M.m.f.}}{l},$$

where l is the length in *cms.* of the double air-gap,—the m.m.f. taken up by the cores being negligible. Thus through the closed circuit in which is impressed the m.m.f. of 566 ampere-turns the flux density at the gap will be

$$B = 283 \text{ maxwells per } sq.\ cm.$$

* See note in Appendix, sec. 2.

Corresponding values of flux density have been determined for the other closed circuits and their ampere-turns given in the above table, and the results have been platted to locate the wave labelled Rotating Flux, in Fig. 109. The wave is drawn here in amount and position corresponding to a single instant $t = 4$. As time progresses the m.m.f.s that establish the flux wave do not change in value nor distribution, but move progressively forward with time as shown in the table of Fig. 110. As the m.m.f.s of each polar interval move forward the flux waves they establish move with them simultaneously. This progression circumferentially with time keeps up uniformly until the entire circumference of the cylinder has been traversed by each wave. Then the process repeats itself indefinitely so long as the nine-phased currents are maintained through the inductors.

Thus it is seen that by means of a static structure a rotating magnetic field having six poles is formed having the same form and character as the rotating field established in the mechanically operated model as described in connection with Figs. 103 and 107.

A study of this wave shows that it obeys the sine law and occupies a position in lagging quadrature with respect to the wave of inductor-currents. The wave of m.m.f. that establishes it has for its values the sums of the corresponding ampere-inductors taken in the manner shown above. From the characteristics of the rotating field formed by polyphase currents as above considered the following properties of the general case wherein the rotating cylinder field is formed by any number of polyphase currents having a symmetrical phase and inductor displacement may now be determined.

Let $I_{max.}$ be the maximum value of each of the polyphase currents;

n, their number;

a, the number of times each circuit traverses the core circumferentially;

f, the frequency of the currents;

l, the single thickness of the cylindrical air-gap; and

B, the flux density of the rotating field formed in the air-gap.

In the above example each circuit ended after traversing the circumference of the core once. These circuits might have been made to traverse the core two or more times, each time going over the same circumferential route as the first when displaced by one inductor space. Denoting the number of times the circuit has thus been applied by a as stated above, it follows that the number of inductors in a positive or negative collection of inductor-currents

$$I_{nd} = an,$$

where I_{nd} is the number of inductors.

The maximum value of the wave of m.m.f. applied per double air-gap, neglecting the reluctance of the iron, will be

$$H = \frac{4\pi}{10} \cdot an \cdot \frac{2}{\pi} \cdot I_{\text{max.}}$$

In this expression, by Sec. 10*e*,

$$\frac{2}{\pi} I_{\text{max.}}$$

is the average value of the current in any set of positive or negative inductor-currents. The sum of such must be

$$\frac{2}{\pi} an I_{\text{max.}}$$

and

This is also the value of the m.m.f. of each set in ampere-turns.

It follows, therefore, that the maximum m.m.f. in gilberts of positive or negative sets of inductor-currents is

$$H = \frac{4\pi}{10} an \frac{2}{\pi} I_{\text{max.}} = \frac{4}{5} anI_{\text{max.}} \quad . \quad . \quad . \quad . \quad (87)$$

The maximum density of the established rotating flux wave will be

$$\textit{Rotating } B_{\text{max.}} = \frac{H}{2l} = \frac{2}{5l} anI_{\text{max.}} \quad . \quad . \quad . \quad (88)$$

Since the flux has a sine wave distribution, by Sec. 10*e*, the average value of the flux density established will be

$$\textit{Rotating } B_{\text{av.}} = \frac{2}{\pi} (\text{rotating } B_{\text{max.}}). \quad . \quad . \quad (89)$$

b. Terminology of Polyphase Currents Applied for the Production of Rotating Cylinder Fields.—In the above example nine phases were used to produce a rotating field. The inductors were spaced apart *one eighteenth of a polar interval or flux wave length.* The currents had a neighboring phase difference of one eighteenth of a period, or 20°. It appears, then, that there were applied here nine phases having neighboring phase differences of one eighteenth period. It is customary to call such a polyphase aggregation of currents a **nine-phase current** or an **eighteenth-phased current.**

The term "nine-phase current" simply implies that nine circuits, each having an alternating current differing in phase position from the others, are employed. The term "eighteenth-phased current" states that the associated currents have an interval phase difference of one eighteenth period, leaving one to infer from general knowledge that nine phases only are used.

The reason why nine phases are used in lieu of eighteen phases with an eighteenth period and polar interval spacing is

due to the following fact: Any wave of current is the equivalent of a wave of equal magnitude one half period or 180° remote from it in phase position, provided such current is sent through the circuit under consideration in a negative or reverse direction. This is easily accomplished by reversing the terminals of that part of the circuit which is to receive the current in a reverse direction. In the above rotating cylinder-field example this was done by running the circuits through their inductor portions in directions alternately positive and then negative, thus in effect applying an eighteen-phase current by means of a nine-phase current.

As will be seen in a later chapter the term *n-phase current* does not always mean that the currents have the *one 2nth* period of phase displacement. Often, as where $n = 3$, the term three-phase is apt to refer to three equal alternating quantities having a *one 3d* period of phase displacement. The term *n-phase*, therefore, in the terminology of the present time simply implies that a polyphase circuit has n circuits wherein the currents may have a neighboring phase difference of *two nth* or *one nth* period.

On the other hand the term "*nth-phased current*," *e.m.f.*, *etc.*, has a perfectly definite meaning. It means that *one-half n* circuits and alternating quantities are employed having a phase and position displacement of *one nth* period or polar interval.

36. Practical Production of Rotating Magnetic Cylinder Fields. — *a. Limited Number of Phases Employed.* — The requirements for simplicity in practice limit the number of phases employed to *two* and *three*, with occasional applications of *four* and *six*. While the higher number of phases are more desirable for the production of rotating cylinder fields in alternating current motors and similar machinery, yet the extra

complication due to the use of so many circuits has limited the number of phases employed as stated above.

b. The Cylinder Fields Formed by Two- and Three-phase Currents Fluctuate.—In the above illustration of the formation of a rotating cylinder field by means of nine phases, so large a number was chosen in order that the changes in m.m.f. from inductor to inductor in each set of positive or negative inductor-currents might not be excessive. Under such circumstances the sum of the inductor-currents and, therefore, ampere-turns of each set remains approximately constant from instant to instant. The rotating flux that is thus established remains likewise approximately constant. The larger the number of phases employed the more exactly constant is the established rotating field.

Where but two and three phases are employed the change in m.m.f. is excessive from inductor to inductor. The total m.m.f. of each set of inductor-currents fluctuates from instant to instant and the wave of rotating flux does not have the sine form; it has a variable form and a fluctuating value.

c. Rotating Cylinder Field Produced by Two-phase Currents.—In the upper portion of Fig. 111 is given a diagram of the circuits of a two-phase arrangement of inductors for the production of a rotating cylinder field. Under all circumstances a similar arrangement of inductors and their connecting circuits used in any class of electrical machinery is called a **winding**.

In the winding of Fig. 111 each of the two-phase circuits traverse the circumference of the core three times. Thus the inductors of each phase occur in alternate groups of three each on the surface of the core. The first phased current passes in series through inductors

1, 2, 3, 7, 8, 9, 13, 14, 15, etc.,

and the second phased circuit passes through the remaining inductors,

4, 5, 6, 10, 11, 12, 16, 17, 18, etc.

As the diagram indicates, the currents and inductors occupy the quarter-phased relation similar to the eighteenth-phased

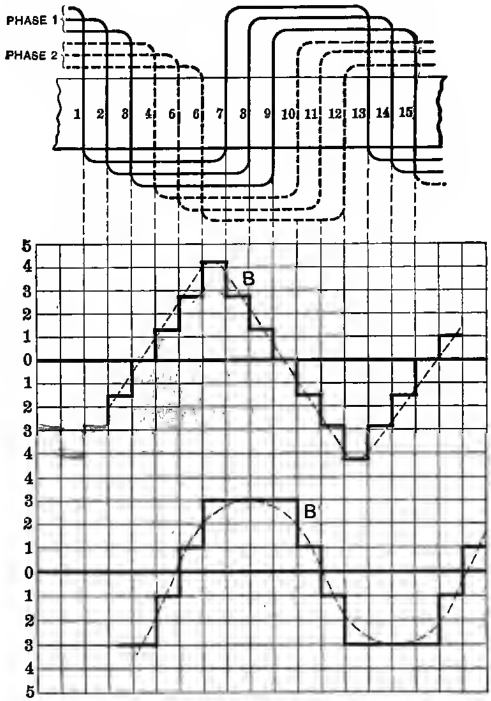


FIG. 111.—Rotating Cylinder Field Formed by a Two-phase Current.

relation used in the above nine-phase illustration. This two-phase winding will establish a rotating cylinder field that will not be entirely uniform in wave form and, therefore, in angular velocity, owing to the fact that the polyphase alternating m.m.f.s, as components of the rotating m.m.f. by which the rotating field is established, are too few in number, causing too

great a transition of m.m.f. values to occur from inductor to inductor.

To bring out the maximum variation in the rotating cylinder field produced by a two-phase current where the inductors are mounted in a winding like that given in Fig. 111, the wave of established magnetic flux may be determined at the extreme limits of such variation. These will occur as follows:

The maximum value of the field flux density occurs when the two-phase currents are equal, and the minimum value when one is zero and the other maximum.

In Fig. 111 the curve B has been plotted from the values of flux established when the two-phase currents are equal, and the curve B' when one has become zero and the other maximum. The reader will have no difficulty in repeating this operation for his own satisfaction, using the method employed in connection with Figs. 109 and 110, and assuming for convenience that the length of the double air-gap is such that the values of B and the m.m.f.s establishing the same will be equal.

The values of the two-phase currents are

$$i_1 = I_{\text{max.}} \sin x, \quad \ldots \ldots \quad (90)$$

$$i_2 = I_{\text{max.}} \sin \left(x - \frac{\pi}{2}\right). \quad \ldots \ldots \quad (91)$$

The flux wave B was traced for

$$I_{\text{max.}} = 1,$$
$$x = 135^\circ,$$

when

$$i_1 = \sin 135^\circ = +.707,$$
$$i_2 = \sin \left(135^\circ - \frac{\pi}{2}\right) = +.707,$$

and the corresponding inductor-current values are

1	2	3	4	5	6	7	8	9	10	11	12 etc.
.707	.707	.707	.707	.707	.707	.707	.707	.707	.707	.707	.707 "

The wave B' was traced when all values had moved forward 45° and

$$i_1 = \sin 180° = 0.0,$$

$$i_2 = \sin\left(180° - \frac{\pi}{2}\right) = +1.0,$$

and the corresponding inductor-current values had changed to

1	2	3	4	5	6	7	8	9	10	11	12	etc.
0	0	0	1	1	1	0	0	0	1	1	1	

The heavy angular lines in these curves locate the flux that would be established if the inductors did not occupy appreciable cross-section. Owing to the fact that their size is always considerable with reference to the depth of the air-gap and the distances between themselves, the resulting wave of flux is always less angular in outline and more like the broken line waves that are drawn in over the angular waves.

By locating the flux waves at other instants they will be found to conform to values intermediate between those given in B and B'.

The maximum values of B and B' are

$$\textit{Rotating } B_{\text{max.}} = 6 \times .707 = 4.242,$$
$$\textit{Rotating } B'_{\text{max.}} = 3 \times 1 = 3.000.$$

Since

$$\text{Sin } 45° = .707 = \frac{1}{\sqrt{2}},$$

their ratio is

$$\frac{B_{\text{max.}}}{B'_{\text{max.}}} = \frac{6}{3\sqrt{2}} = \sqrt{2}.$$

In the general case for the two-phase circuit this ratio is

$$\frac{B_{\text{max.}}}{B'_{\text{max.}}} = \frac{2a}{a\sqrt{2}} = \sqrt{2}.$$

In the rotating cylinder field established by two-phase currents the maximum and minimum values of the rotating m.m.f.s and flux densities have the ratio $\sqrt{2}$.

It is proper at this point to note a comparison of the value of the maximum rotating m.m.f. of a two-phase current set up through the above cylinder winding, as given by the general formula developed in Sec. 35, with those values given by B and B' which are identical with their corresponding ampere-turns.

The value of H in equation (87) is

$$H = \frac{4\pi}{10} \cdot \frac{2}{\pi} anI_{\text{max.}},$$

and in ampere-turns

$$A_t = \frac{2}{\pi} anI_{\text{max.}}.$$

Substituting

$$n = 2,$$
$$a = 3,$$
$$I_{\text{max.}} = 1,$$
$$A_t = \frac{2}{\pi} \times 2 \times 3 = \frac{12}{\pi} = 3.815,$$

while the curves show

$$B = 4.242 \text{ ampere-turns},$$
$$B' = 3 \quad \text{"} \qquad \text{"}$$

The maximum ampere-turn value given by the general equation (87) for polyphase currents falls within the range over which the two-phase maximum ampere-turns or m.m.f.s fluctuate. A comparison of the average ampere-turns as given

by B and B' and equation (89) may also be made. B and B' are averaged from the curves as follows:

$$\left.\begin{array}{l} A_{\text{t. av.}} = B_{\text{t. av.}} = 2.121 \\ A'_{\text{t. av.}} = B'_{\text{t. av.}} = 2.333 \end{array}\right\} \text{Ratio} = .91.$$

$$A_{\text{t. av.}} \text{ by equation (89)} = \frac{12}{\pi} \times \frac{2}{\pi} = 2.430.$$

From these values it is seen that

The average rotating cylinder flux established by a two-phase current fluctuates over a range of ten per cent during each eighth cycle.

*The average rotating cylinder flux thus established is slightly less than the equation for the general polyphase circuit indicates should be the case.**

d. Cylinder Fields Produced by Three-phase Currents.—In the upper portion of Fig. 112 is given a diagram of a three-phase winding that corresponds to the two-phase winding of Fig. 111. The value of a is 2; thus each set of positive or negative inductors will number

$$I_{nd} = an = 6,$$

just the same as applied in the above two-phase case. Thus the winding may employ the same inductors, inductor spacing, and air-gap depth which permits a comparison of the chief features of the rotating cylinder fields formed by the two- and three-phase currents. These are the two forms of polyphase current chiefly used in practice for the production of rotating fields.

* This difference is due to the appreciable distances that must separate the inductors of a two- and a three-phase winding. The effect grows less as the number of phases in use is increased. Stated in another way, the difference is due to the rectangular irregular deviations from the sine form wherein only the outer corners are to be found on the edges of the sine wave to which they belong. With a large number of phases this difference must, therefore, entirely disappear.

For the three-phase circuits the highest of the maximum values of the rotating flux wave occurs when one of the currents is at its maximum and the other two have a value of half

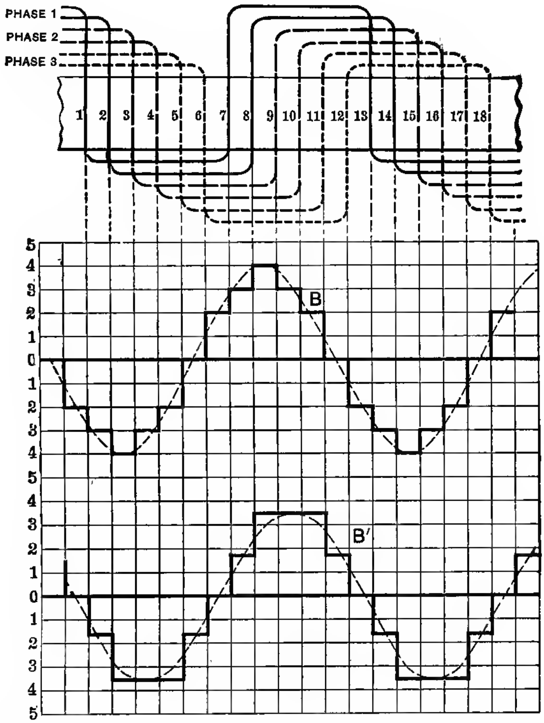


FIG. 112.—Rotating Cylinder Field Formed by a Three-phase Current.

maximum each. The lowest corresponding value occurs when one current is zero and the other two are equal.

The equations of a three-phase current are

$$i' = I_{\text{max.}} \sin x, \quad \ldots \ldots \quad (92)$$

$$i'' = I_{\text{max.}} \sin \left(x - \frac{2\pi}{3}\right), \quad \ldots \ldots \quad (93)$$

$$i''' = I_{\text{max.}} \sin \left(x - \frac{4\pi}{3}\right). \quad \ldots \ldots \quad (94)$$

This is not only a three-phase current but a *third-phased* current as well, and as such is not generally used to produce rotating cylinder fields.

The current i''' is shifted in phase by π, or 180°, by simply reversing its connection in the circuit where it is applied. Thus i''' becomes

$$-i''' = i_2 = -I_{\text{max.}} \sin\left(x - \frac{4\pi}{3}\right), \quad . \quad . \quad . \quad (95)$$

$$= I_{\text{max.}} \sin\left(x - \frac{\pi}{3}\right); \quad . \quad . \quad . \quad . \quad (96)$$

and by writing

$$i' = i_1,$$
$$i'' = i_3,$$
$$i''' = i_2,$$

the three-phase, one-third-phased current becomes a three-phase, one-sixth-phased current, as given by the equations

$$i_1 = I_{\text{max.}} \sin x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

$$i_2 = I_{\text{max.}} \sin\left(x - \frac{\pi}{3}\right), \quad . \quad . \quad . \quad . \quad . \quad (98)$$

$$i_3 = I_{\text{max.}} \sin\left(x - \frac{2\pi}{3}\right). \quad . \quad . \quad . \quad . \quad . \quad (99)$$

In the winding diagram of Fig. 112 the corresponding phases, circuits, and inductors belong together as follows:

No.	Inductors.							
1......	1,	7,	13,	etc.;	2,	8,	14,	etc.
2......	3,	9,	15,	etc.;	4,	10,	16,	etc.
3......	5,	11,	17.	etc.;	6,	12,	18,	etc.

An inspection of equations (97), (98), and (99) shows that one of the values will be maximum and the other two each a half maximum when

$$x = 210°,$$

which will locate B in a convenient part of Fig. 112. Assume

$$I_{\max.} = 1,$$

when the values of the three currents at this instant will be

$$i_1 = \sin 210^\circ = -.5,$$

$$i_2 = \sin\left(210^\circ - \frac{\pi}{3}\right) = +.5,$$

$$i_3 = \sin\left(210^\circ - \frac{2\pi}{3}\right) = +1.0,$$

and the inductor-currents will be

Inductors....	1	2	3	4	5	6	7	8	9	10	11	12, etc.
Currents.....	.5	.5	.5	.5	1	1	.5	.5	.5	.5	1	1, etc.

and the wave B in Fig. 112 is laid down by means of these values.

Further inspection of equations (97), (98), and (99) shows that the next instant at which one of the currents will be zero and the other two equal occurs when

$$x = 240^\circ.$$

The values of the currents will then be

$$i_1 = \sin 240^\circ = -.866,$$

$$i_2 = \sin\left(240^\circ - \frac{\pi}{3}\right) = 0.0,$$

$$i_3 = \sin\left(240^\circ - \frac{2\pi}{3}\right) = +.866,$$

and the corresponding inductor currents will be

Inductors	1	2	3	4	5	6	7	8	9	10	11	12, etc.
Currents	.866	.866	0.0	0.0	.866	.866	.866	.866	0.0	0.0	.866	.866

By means of these values the rotating flux wave has been laid down. The air-gap in this instance has a double depth such that values of B and corresponding values of ampere-turns are identical. This is done for convenience only. From these

curves and the general formulæ applicable to this case the following results may now be deduced:

$$\left.\begin{aligned} B_{\text{max.}} &= 4 \\ B'_{\text{max.}} &= 3.464 \end{aligned}\right\} \text{ average } 3.732.$$

$$A_t \text{ max. by eq. (87)} = \frac{2}{\pi} anI_{\text{max.}} = \frac{12}{\pi} = 3.815.$$

$$\begin{aligned} &B_{\text{av.}} &&= 2\ 333, \\ &B'_{\text{av.}} &&= 2.309, \\ &A_t \text{ av. by eq. (89)} &&= 2.430. \end{aligned}$$

From the above result it follows that

The average rotating cylinder flux established by a three-phase current fluctuates over a range of about one per cent during each twelfth cycle.

*The average rotating cylinder flux thus established is slightly less than the equation for the general polyphase circuit indicates should be the case.**

* See foot-note on page 168.

CHAPTER IX.

THE ELECTROSTATIC FIELD.

SYNOPSIS.

37. General Characteristics of the Electrostatic Field.— The electrostatic field exists everywhere about the electric-current circuit. It has little direct practical value beyond its use in electrostatic indicating or measuring instruments. Indirectly, however, it is universally applied; without its service no electric current could be confined and limited to a definite path along a narrow conductor.

The actual physical properties of the electrostatic field were determined as early as those of the magnetic field. There has, however, been much less working experience with the electrostatic field than with the electric current and the magnetic field. The cause of this lack of experience is due to the fact that we make so little direct use of electrostatic actions. The most practical concern that the electrostatic field usually

gives is the failure of a dielectric under electric pressure to withstand the strains that are thereby produced. In the event of such failure the dielectric in which the field is formed is ruptured and a current follows, forming the common phenomenon of arcing.

Prior to the use of high-pressure alternating currents for the economical long-distance transmission of power, little real difficulty was met with in providing insulations in machinery and on the lines to withstand pressures up to 2000 volts. Later, when the attempt was made to use alternating pressures of 10,000 volts and upwards, great insulation difficulties were found and had to be overcome. In overcoming them much experience with the electrostatic field produced by alternating e.m.f.s has been gained.

a. The Field of Electrostatic Flux.—In Fig. 113, *A* and *C* are metal electrodes that are connected to the terminals of some source of e.m.f. These electrodes are rods that terminate

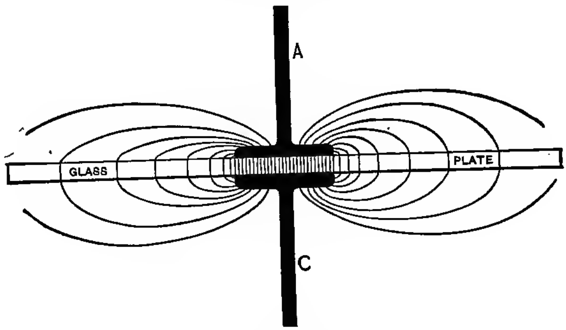


FIG. 113.

in discs between the faces of which is mounted a plate of glass as shown. On applying a moderate electric pressure to *A* and *C* an electrostatic field of flux will be established in density and direction that is shown in the figure by the curved lines. The conventional meaning of these lines thus used to represent the electrostatic field of flux is exactly the same as the corre-

sponding meaning of similar lines that are drawn to represent a magnetic field of flux. In any locality the lines have the direction of the flux, and their number in a unit cross-section, i.e., their rate of occurrence, is proportional to the electrostatic flux density.

Every medium or material through which an electric pressure establishes an electric molecular strain in lieu of an electric current is called a **dielectric**. All non-conductors or insulators are dielectrics.

The electrostatic field is strain produced in the dielectric that everywhere surrounds conductors between which an e.m.f. has produced a difference of potential. The strain is distributed everywhere in such a manner as to require for its application the same potential difference between the two conductors by any route whatsoever. Such a strain in the dielectric media about conductors between which there is a difference of potential constitutes the **electrostatic field of flux.** The quantitative character of its establishment is precisely the same as that for magnetic flux about a permanent magnet or other source of m.m.f. In the production of the strain that constitutes electrostatic flux, a definite amount of current must pass for a definite time. In other and more conventional phraseology, a definite quantity or charge of electricity must be applied to the dielectric.

Simultaneously with the flow of current through a dielectric, a counter-electromotive force is formed in it that is proportional to the time-integral of the current, i.e., the product of the current strength and the time that elapses while such current is passing.

So small is the time-integral of current that is ordinarily necessary for the production in the dielectric of a counter-e.m.f. that is equal and opposite to the impressed or actuating e.m.f., that this process for most practical purposes may be assumed to

occur instantaneously. On the formation of a counter-e.m.f. which is equal and opposite to the impressed e.m.f., the dielectric ceases completely to carry current. Of so little importance and so insignificant is this phenomenon in connection with all ordinary electric circuits at ordinary e.m.f.s that the convention from the beginning of electrical science has been to say that no current passes through a dielectric when it is subjected to an electric pressure. In dealing with condensers it is the general practice even to-day to adopt the convention that electricity is some *thing* instead of a form of energy. For example, we say a condenser has been charged, and in speaking further of the process of charging a condenser we say that a positive charge was taken to one face of the dielectric of the condenser and from the other face an equal negative charge was removed. The real fact to appreciate is that a dielectric subjected to a difference of potential passes no current only after a time-integral of current has passed sufficient to cause the dielectric to produce a counter-e.m.f. equal to the e.m.f. which gives rise to the difference of potential. In forming this counter-e.m.f. a strain or electrostatic flux is produced in the dielectric. This in any case may eventually be made high enough to cause it to rupture.

In establishing electrostatic flux, energy is used in an amount equal to the integral product of the current through the dielectric, the counter-e.m.f., and the time. By uniting the faces of the dielectric with a conductor this energy will be returned by the dielectric and will be dissipated as heat in the conductor by the current that is established. The amount of energy thus returned is never quite equal to the original amount stored, owing to lack of perfect elasticity in the dielectric.

The chief characteristics of the electrostatic field are as follows:

An e.m.f. will pass current through a dielectric until there

is formed in the dielectric an equal and opposite e.m.f. This results in the formation of a field of electrostatic flux within the dielectric.

There exists an electrostatic-flux circuit analogous to the magnetic-flux circuit while the impressed e.m.f. is changing.

While the impressed e.mf. is changing the electrostatic circuit is always closed in a manner entirely analogous to the closed magnetic circuit. The magnetic circuit is made up of magnetic flux and magnetic induction through magnetic bodies. The electrostatic circuit is completed through dielectrics by electrostatic flux that is changing, and by an electric current through a conductor. It follows, then, that an electric current in one part of an electrostatic circuit is the equivalent of a definite rate of change of electrostatic flux in another part of the circuit. The analogy between the electrostatic circuit and the magnetic circuit is not complete in one important respect. It does not hold when the electrostatic flux is not changing. The reason for this failure when the electrostatic flux is not changing is due to the fact that in magnetism there is nothing to take the place of the conductor in electrostatics. That which would correspond to a conductor in magnetism would have to be a medium possessing nearly perfect permeability.*

The analogy is not complete in many minor respects. For example, in the magnetic circuit when the m.m.f. is removed the field of magnetic flux simultaneously ceases to exist. The energy stored in the field in the form of magnetic flux is transformed to some other form in the process of removing the m.m.f. In the electrostatic circuit when the e.m.f. is removed without removing the conducting portion of the circuit in which

* The electrostatic circuit that is analogous to the magnetic circuit is considered here as being an electrostatic field wherein the electrostatic flux density is changing. The present state of the science does not clearly define a further analogy between them. In most respects the electrostatic circuit should be understood to possess characteristics distinctly its own.

the e.m.f. was produced, the electrostatic field of flux will disappear simultaneously and the energy of which it is constituted will be transformed just as in the disappearance of the magnetic field. If, however, we remove both the conductor and the e.m.f. at the same time the electrostatic flux will not disappear from the dielectric. It will remain as a circuit, partially closed upon itself. It cannot disappear, as there is no operating means or vehicle for the transformation of its energy.

The electrostatic field is everywhere manifested through a dielectric, that is, a non-conducting medium. The air is the universal dielectric within which all conductors are ordinarily immersed. The air is often, for mechanical or other reasons, displaced in the immediate neighborhood of the conductors by solid or liquid dielectrics.

Solid and liquid dielectrics are more permeable for electrostatic flux than is air, or than are the gases generally. That is, at a given e.m.f. impressed upon a given thickness and volume a greater amount of energy is stored in solid and liquid dielectrics than in gaseous dielectrics.

b. Specific Inductive Capacity, *K*, is the ratio of the electrostatic flux that is established through a dielectric to that established through air at corresponding dimensions and e.m.f.s.

For convenience, hereafter "electrostatic flux through a dielectric" will be referred to as dielectric flux.

c. The Dielectric Flux Constant, *k*, is the number of coulombs that are passed through an inch cube of the dielectric at a pressure of one volt between two opposite faces.

The **energy** *taken up by a one-inch cube of* **air** *at one volt impressed between two opposite faces is* 1.122×10^{-13} *joules.*

Since a joule is one coulomb-volt, and the average pressure applied in establishing an electrostatic field of flux is *one half* the final pressure, it follows that the number of coulombs

applied per inch cube of air per volt is twice the corresponding amount of energy in joules. This makes

The Dielectric Flux Constant, k, for air as defined above is equal to 2.244×10^{-13} *coulombs.*

In any case the joules of energy, w, stored per cubic inch of dielectric will be

$$w = \tfrac{1}{2}kE^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

where E is the pressure applied per inch of thickness of the dielectric.

The corresponding coulombs of dielectric flux will be

$$D = kE. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

d. The Electric Pressure Gradient, G, is the e.m.f. applied per unit thickness and, therefore, per inch of the dielectric.

Based on the above definitions, the following tables of dielectric properties will be found to be useful:

Name of Dielectric.	K Specific Inductive Capacity.	k Dielectric Flux Constant for Inch Cube Volt,—to be multiplied by 10^{-13}.	w Joules of Energy stored per Inch Cube at one Volt per inch,—to be multiplied by 10^{-13}.
Vacuum	.9996	2.236	1.118
Air	1	2.244	1.122
Glass, hard, old	6.96	15.6	7.8
Glass, hard, new	3.11	6.98	3.49
Glass, hard, 13 m.	3.31	7.44	3.72
Glass, extra dense flint	9.9	22.25	11.12
Glass, lowest value	2.8	6.30	3.15
Porcelain	4.4	9.9	4.95
Shellac	2.74	6.14	3.07
Sulphur	3	6.75	3.37
Rubber, pure	2.12	4.75	2.37
Rubber, vulcanized	2.69	6.04	3.02
Rubber, hard	2.25	5.05	2.52
Paraffin	2	4.50	2.25
Wax	1.86	4.17	2.08
Olive oil	3.08	6.93	3.46
Turpentine	2.25	5.05	2.52
Sperm oil	3.09	6.95	3.47
Petroleum, crude	2.07	4.52	2.26
Petroleum, "headlight"	2.11	4.75	2.37
Vaseline oil	2.17	4.87	2.43

Name of Dielectric.	G Pressure Gradient at Rupturing Point in Kilo-volts per inch.	Name of Dielectric.	G Pressure Gradient at Rupturing Point in Kilo-volts per inch.
Air less than .05 inch...	250	Linseed oil, boiled.....	200
Air .5 to 1 inch........	28	Turpentine oil.........	160
Air 1 to 2 inches.......	25	Copal varnish.........	76
Air 2 to 5 inches.......	18.8	Lub. oil, crude........	46
Air 5 to 10 inches......	15	Vulcabeston...........	91
Glass, clear............	300	Linseed-oil paper......	1000
Mica..................	800	Linseed-oil cloth.......	500
Paraffined paper........	860	Micanite cloth.........	440
Paraffin, melted........	200	Micanite paper........	325

38. The Electrostatic Corona.* — *a. Experiments that Illustrate and Define Corona Phenomena.*—Referring again to Fig. 113, when the glass plate or similar dielectric is about one-tenth of an inch thick and alternating pressure is applied to the electrodes, no easily discernible action occurs until an effective pressure of seven thousand or more volts, dependent upon the properties of the glass, are applied. Then there appears a pale violet light, called the **corona**, at the edges of the electrodes. On raising the pressure somewhat the corona broadens and brightens very much. Then, when the pressure is elevated more, the corona is further increased in amount and brilliancy and thin bright streamers appear. They emanate from the electrodes as a part of the corona, darting variously in all directions. They are shortest and brightest at the electrodes, and taper as they extend outward until they disappear in the more homogeneous corona effect by which they are terminated. On elevating the pressure to from 20,000 to 25,000 volts the streamers extend over large portions of the glass plate. A conventional method is used in Fig. 114 to illustrate this phenomenon. The figure is intended to convey the appearance of one side of the glass plate at one instant.

* Steinmetz, Trans. A. I. E. E., Vol. X., p. 85, 1893.

The electrode and streamers are drawn in solid black, and the homogeneous corona, wherever it is seen to develop, is drawn by means of the broken-line shading. In looking at this phenomenon the eye sees many more streamers at any one instant than really exist. The streamers form and die out with each alternation of the electric pressure. Persistence of vision causes one to see many more streamers at any one instant than actually occur. In Fig. 114 the streamers and corona are

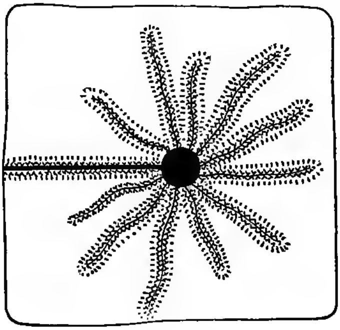


FIG. 114.

drawn much as they are seen to exist at any one instant. The size of the plate is about 8 × 10 inches. Occasionally a pair of streamers, one on either side, will build out to the edge of the plate. They will unite, forming for the instant a short circuit, in a thick, very bright straight streamer that emits a loud report. On raising the pressure to 30,000 alternating volts many of these longer streamers short-circuit over the edges of the plate, emitting a series of loud reports. Finally some one streamer will get over the edge at a sufficiently early stage in an alternation so as to produce heat enough in the short-circuit path to form an alternating-current arc. Such an arc places a continuous short-circuit between the electrodes and, therefore, upon the source. This is promptly stopped by the circuit-breaker in the primary of the high-pressure trans-

former. The corona and the non-short-circuiting streamers also emit sound,—a complex note that corresponds to the alternating periodicity.

If the pressure between the electrodes is maintained at 20,000 or 25,000 volts for a few minutes, generally less than five minutes, the glass plate will puncture. On examining the puncture critically it is observed to be a small hole, melted rather than smashed through the glass. The glass has been heated very much. The points on the glass in the immediate neighborhood of the electrodes upon which streamers strike continuously become highly heated in a short time. One is brought to the conclusion that these streamers are very hot. By operating at a lower pressure where the homogeneous corona is formed without the streamers, it will be seen that such corona develops also a great deal of heat. It is easily discernible, however, that the heat of the streamers is enormously localized as compared with that of the uniform corona.

If an effort is made to apply a pressure between the electrodes that will be high enough to break through the glass before appreciable heating has taken place, one is astonished at the enormous pressure that can be applied for a mere instant to a plate of glass of this thickness. In fact, it is quite impossible, without the use of a very large plate, to apply sufficient pressure to break down this thickness of glass immediately.

That the corona streamers, when the pressure is great enough to start them, will travel long distances, and carry thereby the electric pressure over large areas of the glass or other dielectric, is well brought out in the experiment illustrated in Fig. 115. There a .1-inch glass plate is coated with tinfoil on one side, and on the other a thin wire is wound back and forth at intervals that average $1\frac{1}{2}$ inches. Here and there the intervals amount to double this, or 3 inches, and occa-

sionally the interval is but $\frac{1}{2}$ inch. An alternating pressure of 17,500 effective volts is now applied between the wire and the tin-foil coating. Corona and streamers will form over the entire surface of the plate in a most prolific fashion. A loud complex note is emitted, and the phenomenon presents in variegated fashion great brilliancy. The current used will be about .1 ampere at a power factor of about 50 per cent. In

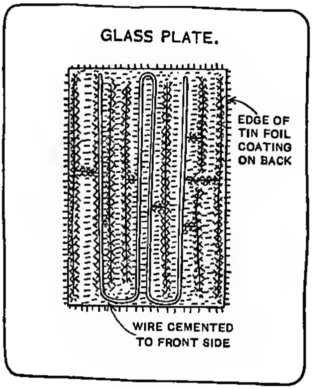


FIG. 115.

this form of apparatus the streamers, though very intense, will strike first from one point and then from another. The plate is not excessively heated at one spot and the operation can be continued for a half-hour without danger of puncturing. Thus one is enabled to determine the curves of e.m.f. and current. Particularly from a study of the curves of current that flow to supply the streamers and the corona, one finds that these streamers form a capacity and air-rupturing phenomenon. When the dielectric flux density at the edges of the electrode, through the little strata of air that exist there, attains a value that will rupture the air, such rupture promptly occurs and the alternating dielectric flux at this point in the air is replaced by an electric current conducted by the arcing phe-

nomenon. Thus in effect the edge of the electrode is extended. On the edge of this arcing current the dielectric flux density is raised also to the rupturing point, and the corona extends further until the fall of potential through it so lowers the flux density that the air beyond is no longer ruptured. The temperature coefficient of the corona as a conductor is negative. As the corona broadens, the least resistance for the passage of current to its outlying districts will occur by the concentration of current along narrow paths that become highly heated and very conductive.

In the light of this behavior one should study the phenomena of corona and streamers on the plate in Fig. 115. The bright streamers are drawn in black and the non-streamer or even corona is drawn in broken-line hatching. It is observed that where the space to be covered is narrow, one-half inch or thereabouts, no streamers form. Where the space is somewhat wider even corona forms along the wires outward to a certain distance. The space along the middle is supplied with a streamer that starts from one point on the wire on one side of the space. This streamer darts to the center of the space and there divides and extends either way along the entire length of the middle space. At the pressure of 17,500 volts here used these streamers will easily extend a foot or more. At a space that is still wider, about three inches, the middle portion will be supplied with even corona that is fed by two streamers, one from the wire on one side and the other from the wire on the other side. In all cases it is clear that they are much better conducting avenues of current for supplying the even corona than the corona itself. They obtain this better conductivity through the heat generated by the current they conduct, owing to the negative temperature coefficient of the highly heated air as a conductor.

That the streamers form easily when they have their origin

from the surface of a metallic electrode, and that they form with great difficulty when such electrode is covered with a dielectric, is brought out by the experiments with the wine-glasses in Figs. 116 and 117 and the glass tubes in Figs. 118 and 119.

The wine-glasses are plain in pattern and differ in size so that one may be set into the other with bottoms touching as shown in Fig. 116. Pressure between the electrodes is turned

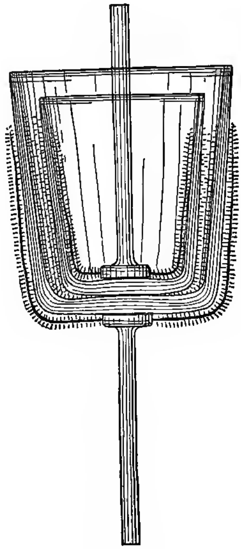


FIG. 116.

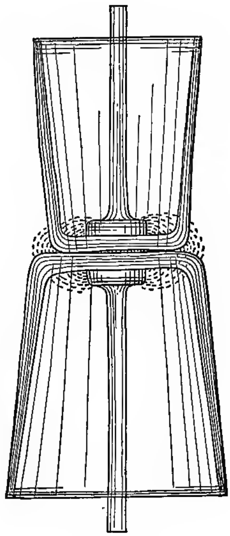


FIG. 117.

on gradually up to 30,000 volts. First the violet blue even corona appears, then after that broadens, the streamers appear and develop in length and intensity until at 30,000 volts they leap over the edges of the glasses, forming a short circuit, and all action ceases on the opening of the circuit-breaker. The corona and streamers are drawn, using the same convention as before.

The glasses are now remounted, as shown in Fig. 117,

and the pressure of 30,000 volts again turned on. Even corona only is now observed. The pressure may be elevated to 40,000 and 50,000 volts without the formation of streamers. There is no change in thickness of the glass dielectric in this experiment. The amount of even corona in the second arrangement in the immediate neighborhood of the electrodes remains the same as in the first arrangement.

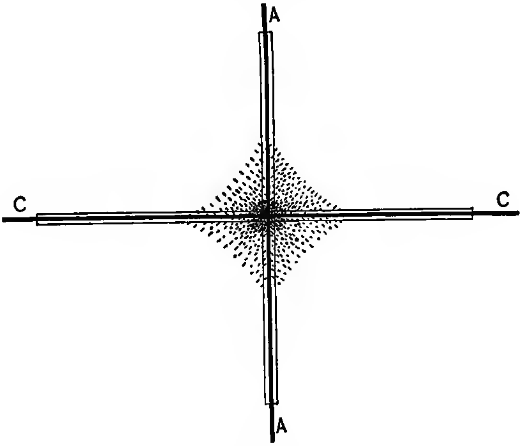


FIG. 118.

In Fig. 118 a 12-inch glass tube having a bore of $\frac{1}{8}$ inch and a thickness of walls of about $\frac{1}{16}$ inch is slipped over an aluminum wire that is just large enough to pass easily through it. At the middle of this tube and at right angles thereto is laid an aluminum wire, the dimensions of which are the same as for the first. A pressure between the wires is now turned on and slowly increased to 20,000 or 25,000 volts. A brilliant display of easily formed streamers will occur. Another glass tube the same as the first is now slipped over the bare aluminum wire and a pressure of 40,000 to 50,000 volts is applied between the wires. A brilliant broad corona forms that is free from all regularly occurring streamers. Occasionally a faint streamer

will run over the surface of one tube or another, in no wise presenting the rich density that the streamers possess in the earlier experiments.

FIG. 119.

In the next experiment the tubes of Fig. 119, which have been described above toward the close of the discussion of Fig. 118, are separated $1\frac{1}{2}$ inches and a pressure of 60,000 volts is applied. There is now witnessed a most beautiful homogeneous corona of strong ultra-violet light. It emits a loud note and rapidly but uniformly heats up the tubes.

b. Testing Dielectrics for Break-down Gradient.—Owing to the intensely localized heating effects of corona streamers and the consequent local destruction of the dielectric, no correct break-down test can be made when they are permitted to form. The homogeneous corona heats so rapidly that a test must be concluded in a very few seconds when such corona is present even in a weak form. Every effort must be made to eliminate the corona as completely as possible when making these tests.

When the samples are of such a character that they may

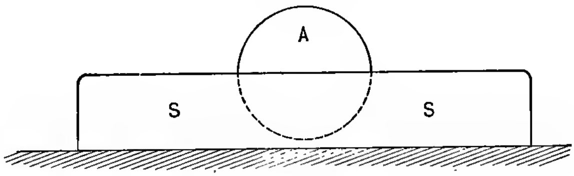


FIG. 120.

be melted and cast into any desired shape, the form of test-piece given in Fig. 120 is excellent. A rectangular slab of

the dielectric is cast about a metal ball as shown. The ball and the metal plate on the under side of the slab will provide the electrodes. The distance between the face of the ball nearest the plate and the plate will be the thickness of dielectric punctured. The lateral thick construction of the sample cuts off all corona.

When the dielectric comes in the form of thin sheets, such as paraffined and linseed-oiled paper, the test samples may be

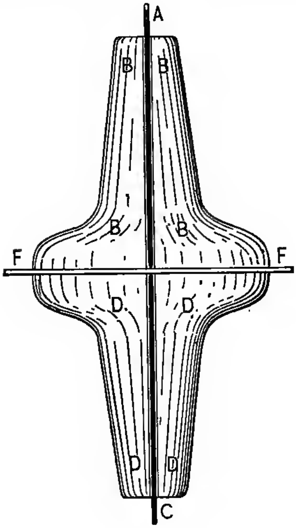


FIG. 121.

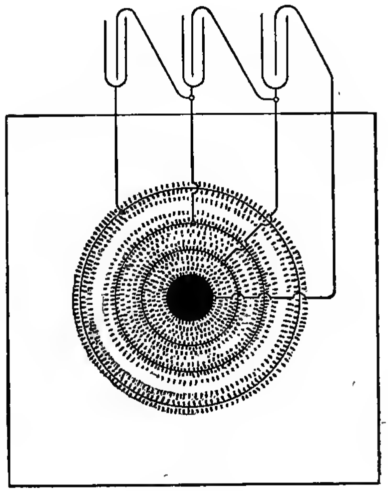


FIG. 122.

made up in pairs of tubes and slipped over thin metal rods mounted at right angles and broken quickly by pressure applied as described in Fig. 118. Glass in the form of tubes may be tested properly by this method.

A liquid sample is easily arranged. The electrode rods, terminated with balls $\frac{1}{4}$ inch in diameter, are separated by a distance through which the liquid is to be broken. In a convenient vessel the liquid is then made to surround the electrodes and the break-down pressure is applied.

Where a strong dielectric sample can be obtained only in the form of a homogeneous sheet it is placed between two glass shields as shown in Fig. 121. Paraffin or other suitable oil is applied and retained in place by capillarity for the purpose of displacing the air and completely avoiding corona. This is fairly satisfactory.

If the sample must be tested as a plate and cannot be tested in the manner described in Fig. 121, the method shown in Fig. 122 may be used. The inevitable corona is kept from concentrating into streamers by the application of the metal guard-rings connected in series by means of condensers as shown. The testing of samples by this method is tedious.

The last method is of practical use either in whole or in part in many other ways where dry insulation is used for building high-pressure apparatus.

The guard-ring or conductor variously applied is a reliable means for modifying the corona and removing many of its most objectionable features.

c. Break-down Test of a Lead-covered Cable.—The insulation and lead cover are removed as shown in Fig. 123. To

FIG. 123.

avoid corona destruction, asbestos wool is applied in a bunch over the ends as shown. Asbestos slightly conducts; thus corona is displaced by the current which the asbestos will carry. In this arrangement cables will, in general, break down at some point under the lead covering instead of at the ends, as will always occur when the corona is permitted.

39. Dielectric Thickness Required to Avoid Corona.—In designing machinery and engineering appliances in which high

pressures are employed, insulators must be selected and arranged in such a manner as to avoid corona almost entirely. The corona must be avoided for two reasons.

(*a*) It is highly destructive. When occurring even to a very moderate degree the insulators or dielectrics over which it is allowed to play suffer rapid deterioration and soon give way. Even the most durable insulators will not last indefinitely under the action of a moderately strong corona.

(*b*) The corona when allowed to form constitutes a serious waste of power. Through economical requirements alone its formation in connection with high-pressure machinery and appliances must largely be eliminated.

The best method for eliminating the corona will be better understood after some study has been made of the character of the corona as brought out by the above experiments and their significance in connection therewith of the varying dielectric properties of the various insulating media or materials as given in the tables on pages 179 and 180.

From the table of rupturing gradients it is seen that thin films of air are ruptured only with much greater pressures proportionally than are greater thicknesses. For example, it requires the application of pressure at the rate of 250,000 volts per inch to rupture an air-film .05 inch thick and but 15,000 volts per inch to rupture air-thickness greater than 5 inches. This property of a rupturing gradient that varies with the thickness belongs in varying degree to the gases only. It is supposed to be due to the fact that metals or conducting electrodes condense on their surfaces dense layers of the gas in which they are immersed. It has been found by experiment that the greater the density and, therefore, the greater the pressure under which a gas exists, the greater the electric pressure per unit thickness that is required to rupture it. From this it follows that if electric conductors do condense the air in

dense films over their surfaces, such films will be much more difficult to rupture electrically than will be the case relatively for much greater thicknesses of air in bulk.

A study of the rupturing gradients of other substances given in this table shows that oils rupture at from about 50,000 to 150,000 volts per inch, and solid dielectrics usually from 200,000 to 500,000 volts per inch, while linseed-oil paper has the exceptionally high rupturing gradient of 1,000,000 volts per inch. It appears, therefore, that even the thinnest films of air are ruptured about as easily as the poorer solid dielectrics, while ordinary thicknesses of air such as are usually employed between conductors upon which high differences of pressure are applied, are grossly inferior in rupturing quality to the average solid dielectric. For air in bulk and for glass the rupturing gradients are 15 and 300 kilo-volts, a ratio of 20 to 1 in favor of glass; for air in bulk and linseed-oil paper the rupturing gradients are 15 and 1000 kilo-volts, a ratio of 66 to 1 in favor of the linseed-oil paper.

Some additional light is thrown upon the practical properties of dielectrics applied to withstand high electric pressures backed up with power when the specific inductive capacity of a dielectric and its rupturing gradient are studied together. By Section 37*c* it was seen that dielectric flux equals

$$D = kE,$$

from which it follows, that since k and E at the point of rupture are constant for any given dielectric of unit thickness, the dielectric flux D at which a given dielectric will rupture is also constant. The value of k is

$$k = 2.244 \times 10^{-13} \cdot K. \quad . \quad . \quad . \quad . \qquad (102)$$

i.e., k and the specific inductive capacity are the same expressed in different units.

As the specific inductive capacities and rupturing pressure-gradients vary widely and independently, a study of the values of dielectric flux should also be made. Obviously such a study must be practically useful where two or more different dielectrics have to be mounted "in series" to withstand the pressure between two conductors. In such a case each dielectric of necessity must be subjected to the same dielectric flux as the others in the series. Under such circumstances the dielectric insulator that breaks at the lowest dielectric flux density will be the first to give way even though individually it may have the highest rupturing pressure-gradient.

The following table gives the rupturing values of dielectric flux for those substances that occur in both of the above tables:

TABLE GIVING RUPTURING VALUES OF DIELECTRIC FLUX DENSITIES IN THE COULOMB-VOLT-INCH SYSTEM OF UNITS.

Air less than .05 inch	$560,000 \times 10^{-13}$
Air .5 to 1 inch	63,000 " "
Air 1 to 2 inches	56,000 " "
Air 2 to 5 inches	42,000 " "
Air 5 to 10 inches	34,000 " "
Glass	3,000,000 " "
Linseed-oil paper,* assumed	5,000,000 " "
Paraffin	900,000 " "
Petroleum	200,000 " "
Turpentine	800,000 " "

A study of this table reveals the fact that while air as a dielectric has many excellent properties, it is unfortunately very

* It is unfortunate that the specific inductive capacities of many of the most important insulators used in practice are as yet unknown in the literature. In assuming $5,000,000 \times 10^{-13}$ as the dielectric flux at which linseed-oil paper breaks down the rupturing pressure-gradient was taken at 1,000,000 volts and its specific inductive capacity was assumed to be 2.25, for which the value of k is 5.05×10^{-13}.

weak in this respect. Except for the thinnest films it breaks at dielectric flux densities far below those at which other common insulators break. Except where a liquid insulator can be used to surround high-pressure conductors and thus completely exclude the air, air always occurs as an insulator inserted in series with other and more powerful dielectrics arranged between the conductors of a high-pressure electric circuit.

It is this unfortunate characteristic of air that gives rise to the phenomenon called corona. In Fig. 113 the flux set up through the glass plate is most dense over the area that is under and directly between the electrodes which face the plate and are pressed against the same. At the edges of the electrodes facing the glass the flux is nearly as great as it is within their surfaces. Under the surfaces of the electrodes, between them and the glass plate, a very thin film of air exists, broken here and there where the electrodes actually touch the glass. Surrounding the immediate edges of the electrodes and separating them and the neighboring surface of the glass plate there are air-films also that are somewhat thicker.

Beyond the edges of the electrodes the flux density rapidly grows less. In a case like this the flux diminishes with the distance from the edges of the electrode more rapidly than does the dielectric strength of the air as its bulk-thickness increases. The result in the case of Fig. 113 is that as the dielectric flux between the electrodes increases with increase of applied pressure a point is reached where the flux amounts to $560{,}000 \times 10^{-13}$, which is great enough to rupture the air-films under and immediately surrounding the edges of the electrodes and the commencement of corona formation occurs.

The limits of this text do not permit analysis of the corona phenomena further than that which is necessary to account

quantitatively for its origin. In the above example of the quantitative formation of corona given in connection with Fig. 113 it was stated that the dielectric flux diminishes more rapidly through the air-space beyond the edges of the electrodes than does the dielectric-flux-density rupturing value for the corresponding air-thicknesses encountered. This accounts for the fact that while air-films that reside on the faces of the electrodes separated by powerful dielectrics have a much higher rupturing gradient than air in bulk, yet they are the first portions of the air to rupture and they are always the seat of initial corona formation. These same circumstances occur in almost all cases where a powerful dielectric is used to insulate two conductors between which is applied a high pressure. This frequently occurs in practice when the conductors must be mounted near or in effect near together in the high-pressure machinery or apparatus. An expression for the thickness of a powerful dielectric that is required to withstand a given pressure without the formation of corona is, therefore, derived as follows:

The dielectric flux at no point between the conductors considered as electrodes must exceed the value above which thin air will be broken, causing the formation of corona. It was seen above that pronounced corona would appear at $560,000 \times 10^{-13}$ units of dielectric flux density in the coulomb-volt-inch system. At about two thirds of this value the corona will be practically absent. The safe dielectric flux density at which corona will be absent and the qualities of the insulating material will not be impaired on its account is $400,000 \times 10^{-13}$. This value in any given case must be equal to

$$\frac{kE}{l} = 400,000 \times 10^{-13},$$

where l is the thickness of the dielectric separating the con-

ductor electrodes, and k and E are the dielectric flux constant of the insulator employed and the applied pressure respectively.

The thickness that a powerful insulator must have for practical duty in withstanding a high pressure backed up with power is, therefore,

$$l = \frac{kE}{400{,}000 \times 10^{-13}}. \quad . \quad . \quad . \quad . \quad (103)$$

40. Dielectric Hysteresis.—It was found in Section 37*c* that energy in definite amounts is required to establish a field of dielectric flux. When the field of flux is destroyed the energy is restored for the most part to the electric energy source that established the dielectric flux originally. This energy is never completely returned to the source; some of it is invariably lost as heat in the medium where the field of dielectric flux is formed. The character of this loss bears a striking analogy to that sustained through magnetic hysteresis, for which reason it has been called **dielectric hysteresis.**

As yet very little is definitely known of the character of dielectric hysteresis beyond the mere fact of its actual existence. One thing is certain, however, that the quantities of this loss ordinarily to be met with are comparatively insignificant and usually negligible. In all practice, as shown in Section 39, insulators must be so selected and proportioned that the dielectric flux set up through them will not be sufficient to cause corona. When the flux and therefore the molecular electric strain of the insulating medium are limited to that extent, it is found that quantitatively dielectric hysteresis is practically negligible. For example, Steinmetz * has found

* Some Notes on Dielectric Losses. Charles Proteus Steinmetz. *Electrical World and Engineer*, Volume XXXVII, p. 1065, June 22, 1901.

that the efficiency of commercial tin-foil-paraffined-paper condensers, manufactured by the General Electric Company, is 99.5 per cent. This means that of the energy supplied to these condensers when operated on alternating-current circuits at normal pressure 99.5 per cent is returned to the circuit by the condenser. A condenser is simply an aggregation of dielectric sheets faced with electrodes that deliver electric pressure which causes dielectric flux to be established through the sheets or plates. Mr. Steinmetz, from a close observation of the phenomena connected with this dielectric loss of .5 per cent, is led to believe that most of it is due to the formation of a weak corona, as may be seen from the closing paragraph of his paper here referred to, and which is quoted as follows:

"As regards the existence of a true dielectric hysteresis, while I believe such a phenomenon exists, I am under the impression that at least a very large part of the observed loss is not due to hysteresis, but is due to traces of air which are still occluded in the dielectric of the condenser; and caused by the mechanical motion of the air-molecules under the influence of the alternating electric stress."

It is important, therefore, to keep in mind the existence of dielectric hysteresis, though under most circumstances that arise in practice the loss of energy occasioned through this phenomenon is of no consequence.

41. Dielectric Conduction.—There are no perfect insulators. This is the same thing as saying that all dielectrics are conductors to some appreciable extent.

This conduction of dielectrics has a twofold character:

(*a*) Current may flow over the surfaces of the dielectric which invariably absorb moisture to some extent and collect dust consisting of conducting bodies.

(*b*) It is found to be a fact that electric pressure will always pass small amounts of current through the bulk cross-section of any insulator or dielectric.

The physical characteristics of the phenomena of dielectric surface current leakage occurring as described above in (*a*) are easily understood. At the present time the nature of actual dielectric conduction as described in (*b*) is not well understood beyond the quantitative facts. It has long since been known that true dielectric conduction, class (*b*), is apt to be least when the dielectric as such is most powerful. For this reason for many years past it has been customary to rate the strength of a dielectric in terms of its **insulation resistance** corresponding to given dimensions of the dielectric. In modern times it has come to be understood that *insulation resistance* is not a reliable factor with which to rate the practical worth of dielectrics as insulators. It has been found that the *dielectric flux constant*, *k*, and the *rupturing gradient*, *G*, are much more rational and trustworthy factors with which to gauge the insulating strength of a given dielectric.

To familiarize the reader with the extent to which ordinary insulators are conductive the following table of insulators and their corresponding *insulation specific resistances* are given: *

Insulator.	Ohms per cm. cube.	Temperature.
Mica................	84×10^{12}	20°
Gutta percha.........	449×10^{12}	24°
Hard rubber..........	28×10^{15}	46°
Paraffin	34×10^{15}	46°
Glass, flint.........	$16,700 \times 10^{15}$	0°
Porcelain...........	540×10^{15}	0°

* These values were compiled from the original sources by Houston and Kennelly. A more complete table may be found in Foster's "Electrical Engineer's Pocket-book."

42. Problems on the Electrostatic Field.—Prob. 62. Determine the capacity, in microfarads, of a glass plate condenser wherein the plate is .1 inch thick and coated on each of the opposite sides with 1 square foot of tin-foil.

Solution: By definition, Sec. 7*d*, and by Secs. 16*c* and 37*c*, capacity in farads equals

$$C = \frac{\Phi_D}{E},$$

where Φ_D is the total dielectric flux through the dielectric.

By equation (101), which is written for unit thickness,

$$\Phi_D = DA = \frac{kEA}{l},$$

where l is the thickness of the dielectric and A its cross-section. It follows that the capacity in microfarads equals

$$C = k \cdot \frac{A}{l} \cdot 10^6.$$

Substituting the dimensions given in the problem and the value of k for old hard glass given in the table on page 179, each in the inch-coulomb system,

$$C = 15.6 \times 10^{-13} \cdot \frac{144}{.1} \cdot 10^6,$$

$$C = .002246 \text{ microfarad.} \quad \textit{Ans.}$$

Prob. 63. It is desired to use vulcanized rubber insulated, lead-covered cable for the electrical connections between a transmission line and the transformers, switches, etc., of a power service. The line pressure is 20,000 volts. What is the minimum permissible thickness that the rubber may have?

Solution: Assume that a single thickness of the rubber may have to endure the full 20,000 volts. The required thickness is obtained by substitution in equation (103):

$$l = \frac{kE}{400{,}000 \times 10^{-13}} = \frac{6 \times 20{,}000}{400{,}000} = .3 \text{ inch.} \quad \textit{Ans.}$$

Prob. 64. What thickness of vulcanized rubber must be used in the above problem if the total pressure is raised to 33,000 volts? .5 inch. *Ans.*

CHAPTER X.

LOSSES IN ELECTRIC CIRCUITS.

SYNOPSIS.

43. Sources of circuit losses.
44. Resistance.
    - *a.* Composition of resistance.
    - *b.* The square mil and the circular mil.
    - *c.* Specific resistance.
    - *d.* Temperature and resistance.
    - *e.* Convenient formulæ.
    - *f.* Problems.
45. Inductance.
    - *a.* Elements of inductance.
    - *b.* Inductance of transmission lines.
    - *c.* Inductance of electric circuit linked with magnetic circuit.
46. Skin effect in conductors.
47. Eddy current losses in conductors.
48. Eddy current losses in magnetic circuits.
    - *a.* Nature of eddy current losses in magnetic circuits.
    - *b.* Eddy current losses in iron wires.
    - *c.* Eddy current losses in iron sheets.
49. Capacity of transmission lines and cables.
    - *a.* Composition of capacity.
    - *b.* Capacity of overhead transmission lines.
    - *c.* Capacity of underground cables.
50. Magnetic and dielectric hysteresis and dielectric conduction.

43. Sources of Circuit Losses.—The transfer of energy through an electric circuit involves certain losses which may be those of current, of e.m.f., or of power, or they may be combinations of these. For example, in the case of a circuit containing inductance and capacity reactance and negligible resistance the loss is one of pressure. In a circuit which contains only distributed capacity, that is, the capacity due to the presence of the conductors and the insulating medium separating them, there is the loss of current used in charging and

discharging the dielectric which separates the conductors. In all practical cases the transfer of electric energy is accompanied by loss of power in one or more of the following ways:

1. In heat produced by the passage of the current along the circuit against resistance.

2. In heat due to the circulation of eddy currents in the conductors.

3. In heat produced by dielectric and magnetic hysteresis in the media about the circuit.

4. In leakage of current through the insulating medium between the conductors.

These losses cover those which are met with in the windings of electrical machinery, such as dynamos, motors, and transformers, as well as in circuits intended for the transmission of electric power to greater or less distances.

The sources of electrical losses (resistance, inductance, and capacity) are properties of circuits which have already been studied from the standpoint of current control. These must now be analyzed in order to determine the quantities which affect their magnitude. Of these properties, resistance may be called that which is inherent in the conductors, while inductance and distributed capacity are due to the relations of the wires forming the circuit to each other, depending also upon the properties of the media separating the conductors. In case capacity is inserted in series with the circuit, its reactance causes a loss of pressure.

44. Resistance.—*a. Composition of Resistance.*—The resistance of a circuit depends upon the dimensions of the conductors and upon their specific resistances, being directly proportional to the length and specific resistance, and inversely proportional to the cross-sectional area. That is,

$$r = \frac{l \cdot r_{sp}}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

For convenience of manufacture, wires are made of certain standard sectional areas. In this country the plan originated by the Brown & Sharpe Manufacturing Company is in general use. In this system, known as the B. & S. gauge, wires range in diameter from No. 0000, having a diameter of .460 inch, to No. 40, having a diameter of .003144 inch. The B. & S. wire gauge is so constructed that successive diameters of wire form very nearly a geometrical progression. The ratio is about 1.123 and the diameters double almost exactly every sixth increase in size. The cross-sectional areas of the wires are almost exactly double every third increase in size. No. 10 wire in this gauge has a diameter of nearly one-tenth inch and a resistance of almost exactly one ohm per thousand feet at ordinary temperature. As other wire gauges are in use, it is usually necessary to specify the gauge in connection with the number of the wire. The properties of both copper and aluminum wire in accordance with the B. & S. gauge are given in the appendix.

b. The Square Mil and the Circular Mil.—In specifying the area of a circular conductor it has become customary to use a conventional unit area based on the linear unit of *one thousandth of an inch.* There is used as an abbreviation for one thousandth of an inch the word **mil.**

Conductors having rectangular cross-sections are estimated and specified in terms of a unit **square mil.**

$$\text{One square mil} = .001^2 \text{ square inch.}$$

Conductors having circular cross-sections are estimated and specified in terms of a unit circular area having a diameter of one thousandth of an inch. This circular unit of area is called the **circular mil.**

$$\text{One circular mil} = \frac{.001^2\pi}{4} = .000{,}000{,}785{,}4 \text{ square inch.}$$

The area of any circle is proportional to the square of its diameter whatever may be the unit of measurement. A circular area measured in circular mils is, therefore, simply the square of the diameter in mils.

In estimating the sizes of round conductors, the use of the circular mil eliminates the repeated use of π. As most conductors are round rather than rectangular, the circular mil is much more frequently used than the square mil.

c. Specific Resistance.—The specific resistance of a conducting material is the resistance of a unit of volume. In conformity with the *c.g.s.* system it is the resistance between two opposite faces of a cube one centimetre on a side. While the *c.g.s.* unit is used to some extent, a more convenient unit is the resistance of one **mil-foot,** that is, of a cylindrical wire one mil in diameter, or one circular mil in area, and one foot long. As one mil-foot of soft copper has a resistance of exactly ten ohms at a temperature of 50.4° F., the resistance of this unit is one easily remembered and applied.

Illustrative Problem.—What is the resistance of a 1,000,000-circular-mil copper cable one mile in length at a temperature of 50.4° F.?

Solution:

$$R = \frac{10 \times l}{\text{cm.}} = \frac{10 \times 5280 \times 1}{1,000,000} = .0528 \text{ ohm.} \qquad \textit{Ans.}$$

d. Temperature and Resistance. — Electrical resistance varies considerably with temperature, each conducting material having its own **temperature-coefficient,** or percentage increase in resistance per degree rise in temperature.

For many conductors this temperature coefficient is not exactly constant. It usually varies slightly with the temperature.

For any particular specimen the variation with temperature is given by the expression

$$R_t = R_0(1 + at + bt^2), \quad . \quad . \quad . \quad . \quad (105)$$

where R_0 is the resistance of such specimen at the standard temperature, and R_t the corresponding resistance at a rise of temperature of t degrees, and a and b are coefficients.

The resistance-temperature coefficients of metals and alloys depend upon their physical condition and the impurities present. As the term bt^2 in the above resistance formula is always small for a given specimen, it is of no practical importance for all ordinary engineering requirements. The variations that occur in degree of purity and physical condition of standard products make this second term useless. The usual practical form of the expression for the temperature-resistance variation of metals and alloys is, therefore,

$$R_t = R_0(1 + at). \quad . \quad . \quad . \quad . \quad . \quad (106)$$

The following table gives the resistance data for the three metals and two alloys most used in electrical engineering for conducting electric currents. Copper and aluminum are used where conduction with minimum loss of electric pressure is desired, and iron, German silver, and manganin are used where great resistance is required.

Material.	Resistance, Ohms per Mil-foot at Zero Centigrade.	Temperature Coefficient, C.°, see eq. (106). a.
Copper, soft, pure......	9.612	0.004284*
Aluminum, pure........	16.02	0.0039
Iron, soft, pure.........	60.	0.00453
German silver..........	127.	0.00044
Manganin.	291.	±0.00001

* This coefficient is important owing to its extensive application for the determination of the working temperatures in electrical machinery. The most authoritative single-valued coefficient is here given. See Report of Committee, *Journal British Institution of Electrical Engineers*, January, 1900, p. 169; Editorial, *Electrical World and Engineer*, March 17, 1900, Vol. XXXV. p. 389.

Weight in pounds of one cir. mil-foot of copper, .000003029.

Iron in addition to its use as a resistance metal is used extensively as a conductor for one or both sides of the electric circuit in electric traction.

German silver is an alloy having 60 per cent of copper, 26 per cent of zinc, and 14 per cent of nickel. It possesses relatively a high specific resistance and a low temperature coefficient. It is extensively used for the construction of resistance appliances for current control in electrical engineering.

Manganin is an alloy, invented by Edward Weston, having 84 per cent copper, 12 per cent manganese, and about 4 per cent nickel. Its temperature coefficient is zero at 45° C.; below that temperature the coefficient is positive, and above negative; in either case it is very small. Manganin is used extensively in the construction of electrical measuring instruments and to some extent for resistances required to take up electric pressure and adjust currents.*

Illustrative Problem.—The resistance of the field circuit of an electric generator not engaged in duty is 100 ohms at the temperature of the surrounding air, 18° C. This generator after being operated for a given time is found to have a field resistance of 109 ohms. The field is wound with copper wire. What rise of temperature causes this increase of field resistance?

Solution: a. The general case is

$$R_{t1} = R_0(1 + at_1),$$
$$R_{t2} = R_0(1 + at_2).$$

Combining and reducing,

$$t_2 = \frac{R_{t2} - R_{t1}}{aR_{t1}} + \frac{t_1 R_{t2}}{R_{t1}}. \quad . \quad . \quad . \quad (107)$$

* Many other metals and alloys are occasionally used in electrical engineering. Extensive data relating to the conducting properties of such metals and alloys are to be found in Foster's Electrical Engineer's Handbook.

Temperature increase $= t_2 - t_1$.

b. The particular case in the above problem. Substituting in equation (107),

$$t_2 = \frac{109 - 100}{.004284 \times 100} + \frac{18 \times 109}{100} = 40.6^\circ \text{ C.}$$

$$\begin{aligned}\text{Temperature increase} &= t_2 - t_1 \\ &= 40.6 - 18 = 22.6^\circ \text{ C.} \quad \textit{Ans.}\end{aligned}$$

e. *Convenient Formulæ*.—These formulæ apply to circular conductors, for which the following symbols will be used:

A, area in circular mils;
A_i, circular mils per ampere;
d, diameter of conductor over insulation in inches;
E, e.m.f. consumed by resistance;
I, current in the conductor;
l, length of conductor in feet;
r, resistance of conductor;
r_{sp}, specific resistance in ohms per circular mil-foot;
w, weight of total conductor in pounds;
w_0, weight in pounds of a circular mil-foot of conductor;
V, volume of a coil in cubic inches.

From equation (104),

$$\text{Ohms of resistance, } r = \frac{r_{sp} \cdot l}{A}.$$

$$\text{Circular mils, cross-section, } A = \frac{r_{sp} \cdot l}{r}. \quad . \quad . \quad . \quad (108)$$

$$\text{Length of conductor in feet, } l = \frac{r \cdot A}{r_{sp}}. \quad . \quad . \quad . \quad (109)$$

$$\text{E.m.f. consumed by resistance, } E = \frac{r_{sp} \cdot l \cdot I}{A}. \quad . \quad (110)$$

Length of conductor in a coil, $l = \dfrac{V}{12 \cdot d^2}$. . . . (111)

Resistance of a coil, $r = \dfrac{r_{sp} \cdot V}{12 \cdot d^2 \cdot A}$. . . . . . (112)

Since $A = A_i \cdot I$, and $l = \dfrac{w}{I \cdot w_0 \cdot A_i}$, by substitution

Watts lost in a conductor by resistance, $I^2 r = \dfrac{r_{sp} \cdot w}{w_0 \cdot A_i^2}$. (113)

f. Problems in the Resistance of Electric Circuits.—Prob. 65. What size of wire (B. & S. gauge) would be selected to transmit 1000 H.P. at 6000 volts terminal pressure a distance of 2.5 miles, with 5.5 per cent loss in the line—(*a*) With copper wire? (*b*) With aluminum wire?

Solution:

$$I = \frac{1000 \times 746}{6000} = 124.3 \text{ amperes.}$$

$$E = 6000 \times .055 = 330 \text{ volts.}$$

(*a*) Substituting in equation (110) and reducing,

$$A = \frac{10.382 \times 2.5 \times 2 \times 5280 \times 124.3}{330}$$

$$= 103,300 \text{ circular mils.}$$

Nearest size B. & S. gauge No. 0 $= 105,534$ circular mils. *Ans.*

(*b*) $$A = \frac{17.303 \times 2.5 \times 2 \times 5280 \times 124.3}{330}$$

$$= 172,100 \text{ circular mils.}$$

Nearest size B. & S. gauge No. 000 $= 167,805$ circular mils. *Ans.*

Prob. 66. What weight of copper wire is required to transmit 10 amperes a distance of 250 feet with 2 volts drop?

No. 6 being the nearest gauge size, the weight is 39.7 lbs. *Ans.*

Prob. 67. What is the loss in watts per mile in a circuit of No. 0 B. & S. aluminum wire when the drop in pressure is 15 volts per mile?

130 watts. *Ans.*

Prob. 68. If a three-phase circuit requires 75 per cent of the copper needed for a single-phase line, at what pressure can 1000 H.P. be delivered 50 miles from the generator by a three-phase current at 16 per cent loss, with the use of 31,576 pounds of copper, neglecting reactance?

31,200 volts. *Ans.*

Prob. 69. In order to produce a given induction in a magnetic circuit a m.m.f. of 1000 ampere-turns is required in the coil which is the source of the m.m.f. The mean length of a turn on the coil is 2 feet. What size of copper wire must be used if 100 volts are supplied at the terminals of the coil? What size if 200 volts are supplied?

Nos. 27 and 30 respectively. *Ans.*

45. Inductance.—*a. Elements of Inductance.*—Inductance has been defined as that property of a circuit by virtue of which a changing current produces within it an e.m.f. of such a nature as to resist the change. This e.m.f. is produced by the increase or decrease of the magnetic field enclosed by the circuit. The process of calculating the inductance of any circuit consists in determining the total change in induction which a given change in the current will produce within the circuit. From the total induction per unit current and the number of turns in the circuit, the voltage corresponding to a unit rate of change of the current can be found. If the permeability of the medium within the circuit is constant, the rate of change of the induction is proportional to that of the current and the inductance is consequently constant. This is the case where the wires are separated by a non-magnetic medium, as in a transmission line suspended in air. If the

circuit produces induction in a magnetic body, the inductance is not constant, as the permeability varies irregularly with the induction.

b. Inductance of Transmission Lines.—A power-transmission line usually consists of a number of wires suspended in air and separated by a distance great enough to insure good electrical insulation. This separation involves the enclosure of such an area that the inductive drop is serious if the line is a number of miles in length.

The density of induction about a conductor has been shown in *c.g.s.* units to be

$$B = \frac{kI}{2\pi r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

(see Sect. 26, page 108). The constant k represents the m.m.f. about a unit current in a straight conductor and is equal to 4π. Equation (114) may be rewritten in the form

$$B = \frac{2I}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

where r is a distance not less than the radius of the conductor, for within the wire the density of flux decreases from a maximum value at the surface to zero at the axis. From the density of flux thus found the total flux which exists about each centimetre length of conductor may be readily determined. The flux between two normal planes, a centimetre apart, will cut the conductor included between the planes when the current disappears from the circuit and will thus generate in it an e.m.f. Over an area dr (see Fig. 124), measured in a plane of the conductor axis, the flux is

$$d\Phi = \frac{2I}{r} dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

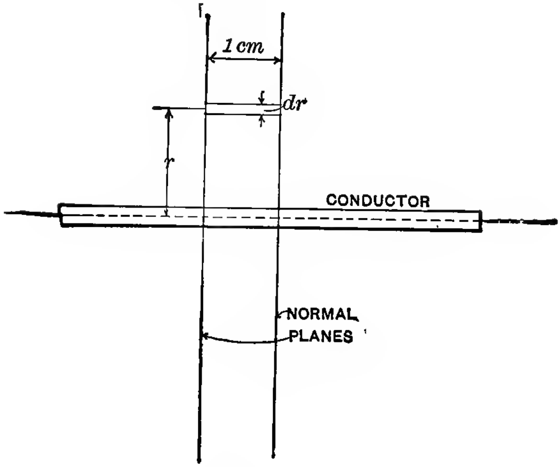


FIG. 124.

The total flux which is included between the conductor and another forming a part of the same circuit and separated from the first by a distance d is

$$\Phi = 2I\int_{r_c}^{d}\frac{dr}{r} = 2I\log_e\frac{d}{r_c}, \quad . \quad . \quad . \quad (117)$$

where r_c is the radius of the conductor.

It is evident from Fig. 125 that the flux produced by the first conductor beyond the second, that is, a greater distance from it than d, has no e.m.f.-producing power, for it cuts both conductors when the field is made or destroyed.

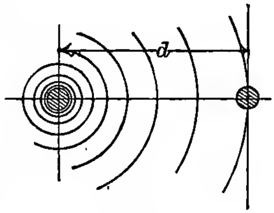


FIG. 125.

From (117) we may write

$$\frac{d\Phi}{dt} = 2\log_e\frac{d}{r_c}\frac{di}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

but

$$\frac{d\Phi}{dt} = E = L\frac{di}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

As inductance has been defined as the e.m.f. produced by a unit rate of change of current in the circuit, if di/dt in formula (119) be made unity, the expression becomes

$$L = 2 \log_e \frac{d}{r_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

The flux within the conductor can be readily calculated by the same method used before, remembering that the flux is less as the conductor axis is approached.

Assuming a uniform distribution of current over the conductor section, it is evident that at any cylindrical surface of radius r the density of flux produced is that due to the current enclosed by the cylinder. Let I be the total current in the conductor. Then

$$B = \frac{r^2}{r_c^2} \times \frac{2I}{r} = \frac{2Ir}{r_c^2}. \quad . \quad . \quad . \quad . \quad . \quad (121)$$

By integration,

$$\Phi = \frac{2I}{r_c^2}\int_0^{r_c} r\, dr = I. \quad . \quad . \quad . \quad . \quad . \quad (122)$$

By reducing to the effect of unit rate of change of current we find that L varies from a value of unity at the axis of the conductor to one of zero at its surface. The average value may be taken at one half. The complete inductance of the conductor is obtained by adding the values within and without it, which gives in *c.g.s.* units

$$L = 2 \log_e \frac{d}{r_c} + \frac{1}{2}. \quad . \quad . \quad . \quad . \quad . \quad (123)$$

The value of the inductance as derived from this formula is of inconvenient size, so that the mile is usually employed as the measure of length and the millihenry (one-thousandth henry) as that of inductance. Inserting the constants for this transformation, (123) becomes

$$L = .160931\left(2 \log_e \frac{d}{r_c} + \frac{1}{2}\right), \quad . \quad . \quad . \quad . \quad (124)$$

where L is the inductance in millihenrys per *conductor-mile.*

The inductance of each of the two parallel conductors of a transmission circuit is the same, hence the total inductance of a single-phase line is twice that of a single conductor.

Prob. 70. What is the inductance of a single-phase transmission line 150 miles long (300 miles of conductor), using wires of .75 inch diameter, suspended 42 inches apart?

.479 henry. *Ans.*

Formula (124) is reliable only when non-magnetic materials are used for alternating-current conductors. If iron or steel be substituted, the inductance is greatly increased. The average inductance *within the conductor* must be multiplied by the permeability of the material to the circular magnetic flux in the wire, and the constant, which is .5 for copper or aluminum, will be about 150 times as great for iron and steel. For this, in common with other reasons, magnetic materials are not used for alternating-current transmission lines.

c. Inductance of an Electric Circuit Linked with a Magnetic Circuit.—When the induction follows a definite path, as in a circuit like that shown in Fig. 126, the inductance, with any particular current flowing, may be readily calculated by the use of permeability or magnetization curves.

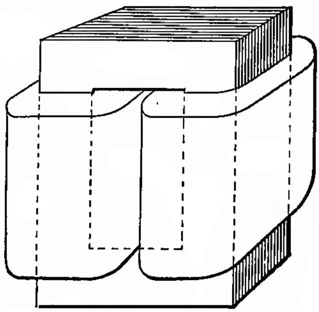


FIG. 126.

By definition of inductance,

$$L = \frac{E}{\frac{di}{dt}}, \qquad (125)$$

$$E \text{ (per turn)} = \frac{d\Phi}{dt},$$

$$E \text{ (total)} \quad = n\frac{d\Phi}{dt},$$

or

$$L = \frac{n\frac{d\Phi}{dt}}{\frac{di}{dt}} = n\frac{d\Phi}{di}. \quad . \quad . \quad . \quad . \quad (126)$$

An inspection of the magnetization curves shows that μ is not constant; therefore for any given current the inductance must be obtained by determining the corresponding flux from the curve. Then (126) may be written

$$L = n\frac{\Phi}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

From equation (83), page 123, in *c.g.s.* units,

$$\Phi = \frac{\mu A}{l} \times 4\pi nI. \quad . \quad . \quad . \quad . \quad . \quad (128)$$

Substituting in (127),

$$L = \frac{4\pi n^2 A \mu}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

where μ corresponds to the particular current at which the inductance is desired.

Prob. 71. Find the inductance of two coils, of 500 turns each, which surround the closed magnetic circuit shown in Fig. 126, when 1 ampere flows through the coils and the constants of the circuit are as follows:

Length of circuit = 4 feet;

Area of circuit = 16 square inches.

The circuit is of electrical sheet steel for which the *B-H* curve is given in Fig. 80, page 117.

Solution: The m.m.f. of 1000 ampere-turns is consumed in 4 feet of circuit, the corresponding ampere-turns per inch being

$$\frac{1000 \times 1}{4 \times 12} = 20.8.$$

The corresponding density in maxwells per square inch is 65,000.

$$\mu = \frac{B}{H} \text{ (in } c.g.s. \text{ units)} = 978.7.$$

$$L = \frac{4\pi \times \overline{1000}^2 \times 16 \times 6.45 \times 978.7}{10^9 \times 48 \times 2.54} = 10.413 \text{ henrys.}$$

Ans.

When the magnetic circuit linked with a coil is made up wholly or partly of air or other non-magnetic material, the reluctance of the path is not definite and experience alone will furnish the necessary judgment for calculating the inductance. The same general principles which give satisfactory results in straight conductors and in closed magnetic circuits will yield results in the other cases which will be accurate in proportion to the accuracy with which the leakage of magnetism between turns and the reluctance of the air-circuit are estimated. This matter is one of design, requiring the use of empirical constants.

46. Skin Effect in Conductors.—The calculations of inductance have been based on the assumption that the current is uniformly distributed in the conductors. This assumption gives satisfactory results in determining the inductance of the circuit as a whole, but the current is really not so distributed. This uneven distribution, while not affecting the inductance of the circuit seriously, does increase the resistance-drop in some cases to a considerable extent, and the cause of this uneven distribution of current, as well as its effect, must now be considered.

Suppose that the conductor is made up of concentric cylindrical layers. The external flux will cut all layers alike, producing no difference in the inductance of the different layers; The outer layer will be cut by none of the flux set up within itself about the axis of the conductor, while the axis will be cut by all flux set up within the outer layer or surface of the con-

ductor. The inductance of the outer layer of the conductor will be due to the cutting of the external flux only, while the inductance at the conductor axis will be due to the cutting of the flux within and without the conductor. Thus it is seen that the inductance of a conductor is greater at its centre than at its surface. The practical result of this is to cause less current to flow at the centre of the conductor and more toward its surface. The reactive e.m.f. at the axis is

$$E' = 2\pi fIL = 2\pi fI\left(2 \log_e \frac{d}{r_c} + 1\right), \quad . \quad . \quad (130)$$

while that at the surface is

$$E'' = 2\pi fI\left(2 \log_e \frac{d}{r_c}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

The resulting difference of e.m.f. is

$$E' - E'' = 2\pi fI. \quad . \quad . \quad . \quad . \quad . \quad (132)$$

The current in the conductor will so distribute itself that the e.m.f. consumed in each cylindrical layer will be the same and therefore the current density will be greatest in the layers with the least counter-e.m.f., that is, in those nearest the surface. The name **skin effect** has been given to this phenomenon. The effect is practically to increase the resistance of the wire by making part of the copper ineffective. It is possible to have such a great counter-e.m.f. at the axis of the conductor as to produce a flow of current in the reverse direction to that at the surface. In this case the resistance drop would actually be less if the centre of the wire were removed, for the reason that the circuit of this reverse current must be completed at the surface of the conductor, and the current density there is greater with the useless current present.

The mathematical calculation of the skin effect is too elaborate to be discussed here, but the following formula will give an idea of the quantities which affect the increase of

resistance met by an alternating current over that met by a direct current:

$$*\frac{R_a}{R_c} = 1 + \frac{1}{12}\left(\frac{2\pi f l}{R_c \cdot 10^9}\right)^2 - \frac{1}{180}\left(\frac{2\pi f l}{R_c \cdot 10^9}\right)^4 +, \qquad . \quad (133)$$

where R_a is the alternating-current resistance;

R_c is the direct-current resistance;

l is the length of the conductor in *cms.*

As $\frac{1}{R_c}$ is a quantity proportional to the area of the conductor, the skin effect is seen to depend upon the area of the conductor and the frequency of the current in it.

This formula applies to non-magnetic conductors. The skin effect in iron and steel is so great as to preclude the use of these metals for economical alternating-current conductors except at low periodicities or small sections or both. It should be noted that the increased drop due to skin effect is an *ohmic* or *resistance* one, and it is not a loss of pressure in reactance, although the latter is present also. The effect of the reactance for the present discussion is to alter the distribution of the current and hence to increase the alternating-current resistance. The skin effect may be reduced by changing the form of the conductor to that of a band, or by making use of several lighter circuits having the same combined current-carrying capacity. At ordinary frequencies and with conductors of moderate size, such as are used for alternating-current transmission, the increased drop is not serious, as is illustrated in the following problem.

Illustrative Example.—How much greater resistance will be met by an alternating current of a frequency of 130 p.p.s. than by a direct current in a No. 0000 copper wire?

Solution: From the wire table we find that No. 0000 has a resistance of .04906 ohm per 1000 feet. After changing

* Formula from Gerard's Leçons sur l'Electricité.

this length to centimetres, substitution in the formula gives as the increase 2.14 per cent. *Ans.*

With larger sizes at this frequency the skin effect rapidly increases, becoming 17.5 per cent for round conductors that are .7622 inch in diameter and carrying current at 133 cycles per second.

47. Eddy-current Losses in Conductors.—In order that conductors may perform their proper function of transferring electrical energy without unnecessary loss, all possible useless currents must be eliminated. Such useless currents heat the conductor, waste energy, and reduce its useful carrying capacity. When a conductor is moving through a non-uniform magnetic field, or is moving into or out of a field, there is a difference in the e.m.f. generated in its different parts. In Fig. 127 the

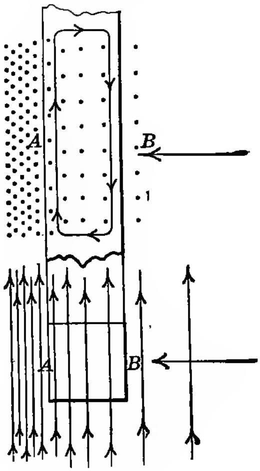


FIG. 127.

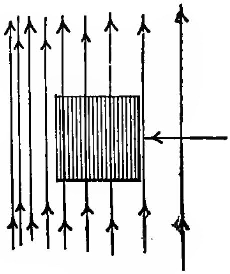


FIG. 128.

conductor is moving from a less to a more dense field so that on the advance side, A, a greater e.m.f. will be produced than on the other side, B. A current sheet is established, and this flows in a direction indicated by the closed arrow line. This current heats the conductor and hence reduces the capacity

of the machinery in which it occurs; it is known as an **eddy current.**

Eddy-current losses may be reduced and practically eliminated by stranding the conductor or by laminating it, the plane of the sheets being perpendicular to the direction of motion. (See Fig. 128.)

Eddy-current losses are not important in small conductors in dynamos and motors, but may reach serious proportions in the large, bar-wound armatures of generators or motors of large capacity if proper precautions are not taken to prevent their occurrence.

48. Eddy-current Losses in Magnetic Circuits.—*a. Nature of Eddy-current Losses in Magnetic Circuits.*—Steel, iron, or other magnetic material that is permeated by an alternating magnetic flux will form about and at right angles to such flux closed conducting circuits in which e.m.f.s are produced and in which eddy currents will be established. Such currents are generally entirely useless, and the I^2R power used in circulating them is completely wasted in heating the magnetic material. The important losses of this character occur where a magnetic circuit has established through it an alternating flux by means of an alternating-current circuit wound about the magnetic circuit, or in electrical machinery where a part of the magnetic circuit is subjected to alternating flux or pulsating flux.

In the first case the electric power in the alternating-current circuit furnishes the energy required to circulate the eddies in the magnetic circuit with which it is linked; these eddy losses thus become a part of the losses of pressure and power losses that occur in the electric circuit.

In the second case mechanical power is the source from which the eddy-current losses are taken. Mechanical power thus applied is a portion of the total mechanical power supplied

to the generator, or in the motor it is a portion of the mechanical power developed. Under these circumstances the eddy-current losses do not form a part of the losses that occur in a particular electric circuit.

The magnitude of these losses is independent of their supply source. It depends upon the alternating flux density, frequency, form of the magnetic material in relation to the direction of the flux, and upon the resistance of the same.

No attempt is made in engineering to use solid magnetic circuits to accommodate alternating flux because of the magnitude of eddy losses occurring therein. Such circuits are invariably built up of thin sheets or thin wires. The surfaces are coated in some manner so as to insulate them from one another. Such insulation may be a coating of black oxide, japan, or asphaltum. Thus the only eddies that are formed in magnetic circuits under practical conditions are those due to the thicknesses of the wires or sheets employed in constructing such magnetic circuits.

b. Eddy-current Losses in Iron Wires.—The circular cross-section of an iron wire is given in Fig. 129. The eddy-current loss in a cylindrical differential portion dA of unit length will first be determined; the total loss per unit length may then be obtained by integration.

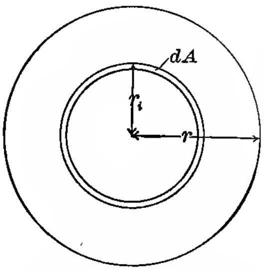


FIG. 129.—Section of Iron Wire.

$B_{max.}$ is the flux density in the wire in max. p. sq. in.;

r is the radius of the wire;

R_{sp} is the resistance of an inch-cube of the wire.

The e.m.f. in the circular portion dA, Fig. 129, from equation (40) equals

Φ_{max}

$$E = (2\pi f)\left(\frac{1}{\sqrt{2}} \cdot \pi \cdot r_i^2 \cdot B_{max.} \cdot 10^{-8}\right),^*$$

* See note in Appendix, sec. 3.

wherein, by Sec. 7*b*,

$E = 2\pi f L I$

$$L = \frac{\pi r_i^2 B_{\text{max.}}}{\sqrt{2} \cdot I \cdot 10^8}.$$

Watts lost per unit length of dA:

$$dw = d(I^2R) = d\frac{E^2}{R}.$$

Resistance per unit length of dA:

$$R = \frac{2\pi r_i R_{sp}}{dr_i},$$

$$dw = \frac{(\sqrt{2}\pi^2 f B_{\text{max.}} r_i^2 10^{-8})^2 dr_i}{2\pi r_i R_{sp}}.$$

Watts lost in wire per unit length:

$$w' = \frac{\pi^3 f^2 B^2_{\text{max.}}}{R_{sp} 10^{16}} \int_0^{r} r_i^3 dr_i,$$

$$w' = \frac{\pi^3 f^2 B^2_{\text{max.}} r^4}{4 R_{sp} 10^{16}}. \qquad (134)$$

Volume of wire per unit length:

$$v = \pi r^2.$$

Length of wire per cubic inch of iron:

$$l = \frac{1}{\pi r^2}.$$

Watts lost per unit-cube of iron wire:

$$w = \frac{w'}{\pi r^2},$$

$$w = \frac{\pi^2 f^2 B^2_{\text{max.}} r^2}{4 R_{sp} 10^{16}}. \qquad (135)$$

R_{sp} per inch-cube reduced from the mil-foot value given on page 203 is

$$R_{sp} = \frac{60}{12 \times 10^6} \cdot \frac{\pi}{4} = \tfrac{5}{4}\pi \times 10^{-6}$$
$$= 3.927 \text{ microhms at } 0.0° \text{ C.}$$

The watts lost through eddies due to alternating magnetic flux in soft iron wire of round cross-section at zero degrees C., neglecting skin effect, per *cu. in.* are therefore

$$W = .63 f^2 B^2_{\text{max.}} r^2 \times 10^{-10}, \quad . \quad . \quad . \quad (136)$$

where r is the radius of the iron in inches,

B is the maximum flux density per square inch and

f is the frequency in cycles per second.

Illustrative Example.—Find how many watts will be lost through eddies per cubic inch of iron wire where the data are as follows:

Radius of wire........................	.1 inch
Flux density, m. p. sq. in............	10,000
Frequency........................	100

Substituting in equation (136),

$W = .63 \times 100^2 \times 10{,}000^2 \times .1^2 \times 10^{-10} = .63$ watt. *Ans.*

When iron wire is used for alternating magnetic cores its diameter should be made small enough to limit the loss through eddy currents to a certain per cent of the hysteresis loss.

Let ϵ be the ratio of eddy to hysteresis losses thus allowed.

Then, from equations (86) and (136),

$$\epsilon \eta B^{1.6}_{\text{max.}} f \times 10^{-7} = .63 f^2 B^2_{\text{max.}} r^2 \times 10^{-10}.$$

$$r^2 = \frac{\epsilon \eta B^{1.6}_{\text{max.}} f \times 10^{-7}}{.63 f^2 B^2_{\text{max.}} \times 10^{-10}},$$

$$r = \sqrt{\frac{1590 \epsilon \eta}{f B^{.4}_{\text{max.}}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

Illustrative Example.—What must the radius of the wire be to keep the eddy losses within 25 per cent of the hysteresis loss of excellent commercial electrical steel wire operated at a flux density of 30,000 maxwells per square inch and 100 cycles?

$$\epsilon = .25,$$
$$\eta = .0015,$$
$$B_{\text{max.}} = 30{,}000,$$
$$f = 100.$$

R_{sp} assumed the same as for pure soft iron. Substituting these values in equation (137),

$$r = \sqrt{\frac{1590 \times .25 \times .0015}{100 \times 30{,}000^{.4}}}.$$

$r = .0098$ in., or a diameter of .0196 in. *Ans.*

c. Eddy-current Losses in Iron Sheets.—Let t be the thickness of the sheet in inches; b the width of the sheet in inches.

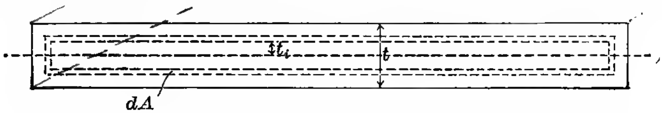


FIG. 130.—Section of Iron Sheet.

E.m.f. generated in the circuit of a differential section dA in Fig. 130 will be

$$E = 2t_i b\sqrt{2}\pi B_{\text{max.}} f \times 10^{-8},$$

where it is assumed that b is large when compared with t. The resistance of the differential section dA per unit length

$$R = \frac{R_{sp}(2b)}{dt_i},$$

$$dw' = \frac{(2b\sqrt{2}\pi B_{\text{max.}} f \times 10^{-8})^2(t_i^2 dt_i)}{2R_{sp}b},$$

$$w = \frac{4b^2 2\pi^2 B^2_{\text{max.}} f^2 \times 10^{-16}}{2R_{sp}b}\int_0^{\frac{1}{2}t} t_i^2 dt_i,$$

$$w = \frac{b\pi^2 f^2 B^2_{\text{max.}} t^3}{6R_{sp}10^{16}}.$$

Volume per unit length of sheet,

$$v = bt.$$

Length per unit cube,

$$l = \frac{1}{bt}.$$

Watts per unit cube,

$$W = \frac{\pi^2 f^2 t^2 B^2_{\text{max.}}}{6R_{sp} \times 10^{16}}. \quad . \quad . \quad . \quad . \quad (138)$$

R_{sp} for iron per inch-cube as determined in above,

3.927 microhms.

Watts lost in iron sheets per inch-cube at 0° C.,

$$W = .4195 t^2 B^2_{\text{max.}} f^2 \times 10^{-10}. \quad . \quad . \quad . \quad (139)$$

The thickness of sheet required to limit the eddy loss to a fraction, ϵ, of the hysteresis loss is

$$t = \sqrt{\frac{2385 \epsilon \eta}{f B^{.4}_{\text{max.}}}}. \quad . \quad . \quad . \quad . \quad . \quad (140)$$

Illustrative Example.—Use the same example as that given above in this connection for wire.

Solution :

$$t = \sqrt{\frac{2385 \times .25 \times .0015}{100 \times 30{,}000^{.4}}}.$$

$$t = .0120 \text{ inch thick.} \quad \textit{Ans.}$$

The corresponding diameter or thickness of wire was determined to be .0196 in., from which it is found that the ratio of the thickness of iron or electrical steel wires to the thickness of iron or electrical steel sheets wherein the same eddy losses occur is

$$\frac{.0196}{.0120} = 1.633.$$

This same result may be obtained by deducing the ratio of the diameter of the wire to the thickness of the sheet for the general case wherein the eddy losses are equal. This is

accomplished by dividing $2r$ by t, as given by equations (137) and (140),

$$\frac{2r}{t} = \sqrt{\frac{4 \times 1590}{2385}} = 1.633. \quad . \quad . \quad . \quad (141)$$

49. Capacity of Transmission Lines and Cables.—(*a*) *Composition of Capacity.*—Capacity in a circuit may be localized, as in a condenser, or it may be distributed along a circuit and be due to the medium between the conductors. If a condenser is in series with a circuit, *e.m.f.* is consumed. When a condenser is in parallel with a circuit, or when the dielectric medium between the conductors provides a distributed capacity in parallel with the circuit, a leading current of low-power factor is drawn from the generator and a certain amount of energy is wasted in the resistance of the circuit due to such *charging current.* In long insulated and armored underground cables this current may be of considerable magnitude. In long air lines it may be great enough to have an influence upon the operation of the line circuit. As the current is practically in leading quadrature with the pressure it will have a tendency to neutralize the magnetizing current of transformers, induction motors, and similar apparatus, and hence in some cases prove a benefit, rather than a detriment.

b. Capacity of Overhead Transmission Lines.—In Fig. 131 C' and C'' are two long conductors mounted in the air remote from other conductors. They are connected to some source of e.m.f. that produces an electric pressure between them which sets up a field of dielectric flux. The field is set up at right angles to the conductors. Its distribution is shown by the curved lines drawn in the figure. From point to point throughout the field the flux is ever varying in amount and direction, due to the fact that the cross-section of the space through which the flux is established constantly varies. The

distribution of flux that actually occurs must be one by which the counter-pressure formed by the presence of the flux taken over any route from conductor to conductor is equal to the

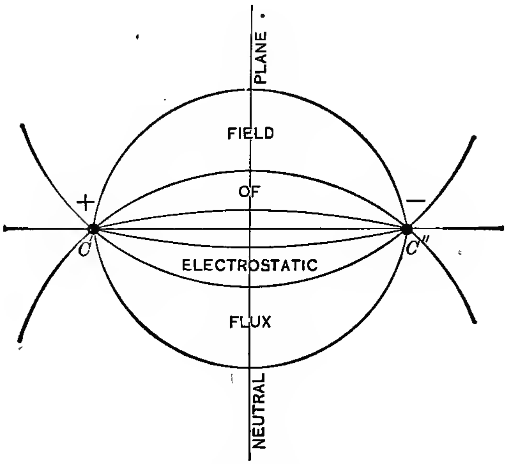


FIG. 131.

applied pressure. By definition, Sec. *7d*, the capacity between any electric conductors equals

$$C = \text{coulombs per volt.}$$

$$C = \frac{\Phi_D}{E}, \quad \ldots \ldots \ldots \quad (142)$$

where Φ_D is the total electrostatic flux as specified on page 175.

By equation (101), which is written for a unit cube,

$$\Phi_D = DA = \frac{kEA}{d},$$

where d is the thickness of the dielectric and A its cross-section. Substituting in equation (142),

$$C = \frac{kA}{d}. \quad \ldots \ldots \ldots \quad (143)$$

The values of A and d are composite for the case given in Fig. 131, so this expression can be applied only to a differen-

tial portion of the dielectric in which the conductors are mounted.

The vertical line drawn midway between the conductors C' and C'' locates a neutral plane. Between no two points of this plane can there be a difference of pressure. A conducting sheet of *great area* may be mounted in the field in this position without disturbing the flux in amount and distribution. One half of the e.m.f. applied between the conductors, $\frac{E}{2}$, may be applied between C' and the plane, and the other half between the plane and C'', without altering the field in amount and direction. It is evident that one of these conductors may now be removed with the result that its corresponding half of the electrostatic field will disappear without in any way altering the other half, and as shown in Fig. 132.

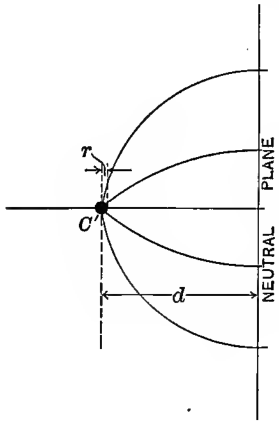


FIG. 132.

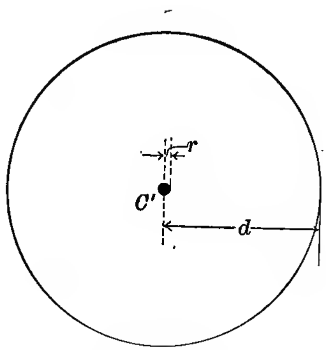


FIG. 133.

Mathematical research has shown that where a medium is permeated by a field of flux emanating from a long round wire to a body presenting a plane surface of indefinite extent, the effective length of the path of such flux is double what it is for the case in which the same wire sets up

a flux through the above medium to the walls of a cylinder of indefinite length, the radius of which is the same as the distance between the wire and plane, with the restriction that the distance of the wire from the plane must be twenty or more times its own diameter. This has been found to be true for any character of flux through any medium wherein the pressure taken up is directly proportional to the flux density and the distance in the direction of the flux. It follows, therefore, that the distance of flow of dielectric flux, magnetic flux, or electric current in the medium surrounding C' in Fig. 132 between it and the plane is double that for the same medium in Fig. 133 between C' and the walls of the cylinder, d and r being the same in both cases.

The conductor C' in Fig. 133 and the conducting cylinder are mounted in the free air and replace the conductor C' and the conducting plane in Fig. 132, also mounted in the free air, with d and r the same in each case, for the purpose of determining the capacity of the conductor C' in coulombs per volt applied between the conductors.

Φ_D is the total dielectric flux in coulombs per inch length of conductor.

D is dielectric flux density at any point within the field.

d_i is any portion of d measured from the center of C'.

Then

$$D = \frac{\Phi_D}{2\pi d_i},$$

likewise

$$D = k\frac{dE}{dd_i}.$$

Therefore

$$k\frac{dE}{dd_i} = \frac{\Phi_D}{2\pi d_i},$$

$$dE = \frac{\Phi_D}{2\pi k}\cdot\frac{dd_i}{d_i}.$$

Integrating,

$$E = \frac{\Phi_D}{2\pi k}\int_r^d \frac{dd_i}{d_i},$$

$$E = \frac{\Phi_D}{2\pi k} \log_e \frac{d}{r}. \quad . \quad . \quad . \quad . \quad (144)$$

Since capacity in farads is $C = \frac{\Phi_D}{E}$,

$$C = 2\pi k \frac{1}{\log_e \frac{d}{r}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

For practical purposes it is convenient to use the common logarithm, the microfarad in lieu of the farad, and the mile for the unit of length.

For air

$$k = 2.244 \times 10^{-18}.$$

Reduction factor for miles.........	63,360
" " " microfarads....	1,000,000
" " " $\log_{10}$..........	2.3026

Making the above substitutions and reductions, the capacity of a single wire becomes

$$C = \frac{.0388}{\log_{10}\left(\frac{2d}{r}\right)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)^*$$

where d is the distance of the centre of the conductor from the neutral plane, and r is the radius of the wire.

The capacity of two parallel wires belonging to the same circuit suspended in the free air will be $\frac{1}{2}C$, since the capacities of each wire to the neutral plane are in series. This may be seen from another point of view.

A current which flows through a conductor to establish dielectric flux is called a **charging current.** By equation (44)

* The effective length of flux path is $2d$. See page 226.

the charging current due to the capacity of a conductor with reference to its neutral plane is

$$I = \frac{2\pi f C E}{10^6}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

From symmetry the electromotive force required to establish the charging current I from wire to wire is $2E$, and

$$I = \frac{2\pi f C'(2E)}{10^6}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

where C' is the capacity from one wire to the other.

$$C' = \frac{10^6 I}{2\pi f(2E)};$$

from (147),

$$C = \frac{10^6 I}{2\pi f E};$$

therefore

$$C' = \tfrac{1}{2}C. \quad . \quad . \quad . \quad . \quad . \quad (149)$$

The capacity from wire to wire of a single transmission line is

$$C = \frac{.0388 l}{2 \log_{10}\left(\frac{2d}{r}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (150)$$

where C is the capacity of the line in microfarads;
l is the length of the line in miles;
d is half the distance between the lines in inches;
r is the radius of the line conductor in inches.

Illustrative Example.—Find the charging current in a transmission line having the following dimensions:

Radius of conductor..........	.1625 inch
Distance between centres	24 inches
Length of line....	50 miles
E.m.f.......................	50,000 volts
Frequency.	60 cycles

Substituting in equations (147) and (150),

$$I = \frac{2\pi \cdot 60 \cdot 50{,}000}{10^6} \times \frac{.0388 \times 50}{2 \log_{10}\left(\frac{24}{.1625}\right)} = 8.47 \text{ amperes. } \quad \textit{Ans.}$$

c. Capacity of Underground Cables.—For underground service the conductor usually has a circular cross-section. It is insulated with a cylindrical sheath of strong dielectric over which a lead covering is drawn to keep out moisture. Over the lead there is generally applied an extra sheathing for mechanical protection.

The value of the capacity given by equation (145) evidently applies also to this case. This value reduces to

$$C = \frac{32 \cdot 772 kl \times 10^6}{\log_{10}\left(\frac{d}{r}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (151)$$

where C is the capacity in microfarads;

k is the dielectric flux constant in coulombs per inch-cube per volt;

l is the length of the cable in feet;

d is inner radius of the lead sheathing;

r is the radius of the conductor.

Illustrative Example.—What is the capacity of a vulcanized-rubber lead-covered cable wherein the radius of the conductor is .25 in. and the radius of the lead sheath .55 in. and the length of the cable is 3 miles?

The value of k for vulcanized rubber is given in the table on page 179 at 6.04×10^{-18}. Substituting these values in equation (151),

$$C = 6.04 \times 10^{-18} \times \frac{32 \cdot 77 \times 10^6 \times 5280 \times 3}{\log_{10} \frac{.55}{.25}},$$

$C = .915$ microfarad. *Ans.*

50. Magnetic and Dielectric Hysteresis and Dielectric Conduction.

In alternating-current circuits electrostatic and magnetic fields are formed about the conductors as the pressure and current attain positive values, and when such pressure and current reverse by passing through zero their electrostatic and magnetic fields are destroyed and then re-formed with an opposite sign corresponding to the change of sign that has taken place in the electric pressure and current. In forming an electrostatic or a magnetic field a certain amount of energy is removed from the circuit. Such energy for the most part is returned to the circuit from which it was derived each time that the field passes through zero. It is never all returned to the circuit, as some energy is always lost in hysteresis. This loss impresses itself upon the electric circuit in the same manner as does a loss due to resistance. The magnitude of magnetic and electrostatic hysteresis losses have been treated in Chapters VII and IX. Magnetic hysteresis losses are large enough in practical operations to require that they be kept in mind and limited as much as possible under most circumstances.

Electrostatic hysteresis from a practical point of view appropriates insignificant amounts of energy. In engineering it is not considered as a factor in the economical operation of machinery and circuits.

Where powerful dielectrics are used to insulate conductors delivering high-pressure current there is some loss of energy by direct conduction of current through the dielectric. (See Sec. 41.) The magnitude of the energy lost in this manner is so small as not to be a factor in electrical-engineering economies, particularly where the dielectric is satisfactory in other respects.

The corona is a gaseous dielectric conduction that is quite wasteful of electrical energy. As stated in Chapter IX, it is

not allowed to form over conductors in electrical machinery that must be brought near together and insulated by powerful dielectrics on account of the rapid destruction of the insulation that it causes. The destructive character of the corona has less opportunity to manifest itself on high-pressure long-distance transmission lines, which are composed of round bare conductors supported in the air on glass or porcelain insulators at intervals of about 100 feet, than in the insulation of high-pressure machinery, transformers, etc. Under such circumstances, however, the extent to which the corona is allowed to form is closely limited also on account of the loss of energy that it involves.

The loss of power in high-pressure transmission circuits by the corona class of dielectric conduction has been examined experimentally and reported upon by eminent engineers.*

In the diagram of Fig. 134 are plotted the losses observed by measurements made by R. D. Mershon upon a line of conductors, .162 inch in diameter and 11,720 feet in length, located at Telluride, Colorado, separated 15, 22, 35, and 52 inches and subjected to alternating pressure varying from 20,000 to 60,000 volts. The curves labelled 15 in., 22 in., 35 in., and 52 in. show the relation between the watts lost on the line and the alternating pressures when the wires were separated by the corresponding distances 15, 22, 35, and 52 inches. The form of the applied pressure wave approximated the sine wave.

Careful experimental work by Mershon developed the facts that the losses recorded by the portions of these curves below the sudden upward bend are due to a general current leakage over the insulators, and to a very small extent through the at-

* High-voltage Power Transmission, by Chas. F. Scott. Trans. Am. Inst. Elec. Eng'rs, Vol. XV, p. 531, 1898.

mosphere; and that the great losses recorded above the bend in the curves are due entirely to corona conduction.

A study of these results shows that the corona conduction of the capacity-charging current of a high-pressure transmission line is very wasteful of electric power. The results show that wide separation of the conductors somewhat increases the pressure at which the loss begins, but does not alter the rate of increase of the loss with increase of pressure.

The curves bring out clearly the fact that there exists a critical pressure for each given set of line conditions, at which a further increase in pressure causes a very much greater proportional loss of power. The cause is obviously as follows:

When an electric pressure is applied between a pair of parallel round conductors mounted in the atmosphere a dielectric flux will be established in the atmosphere about the conductors in amount and distribution as shown in Fig. 131. The dielectric flux density under these circumstances is much greater in atmospheric zones next to the surfaces of the conductors than it is at a greater distance from the conductors. It follows that when the electric pressure applied between the conductors is increased, a point will be reached at which the dielectric flux density in the air next to the conductor surface will exceed the rupturing point and will break through a certain radial distance in the air. When this occurs loss of power at once ensues. Such loss occurs through heat generated by the gaseous conductivity that constitutes the corona. On further increase of the pressure, the ruptured zone through which corona is displayed rapidly increases in depth with correspondingly rapid increase in power consumed. This is due to two causes:

1. When air becomes conductive by dielectric rupture or by the formation of an arc by the separation of the connecting terminals of a circuit carrying current, it presents a resistance to the flow of the current that diminishes as such current

increases. Thus, in the case under consideration, while the diminution of the dielectric flux density with the radial distance from the centre of the conductor tends to limit the thickness of the corona, the inverse resistance-variation character of the corona current-conducting zone tends to extend the thickness of such zone and thus counteract the limiting action of the radial lessening of the flux density.

2. After rupture of the air next to the metallic surface of the conductor, the capacity of the conductor has been increased by the virtual increase in the diameter of the conductor that has occurred through the existence about it of the conducting atmospheric zone. An increase in capacity is followed by an increase in current flow, which, as stated above, reacts to increase the thickness of the corona or conducting zone.

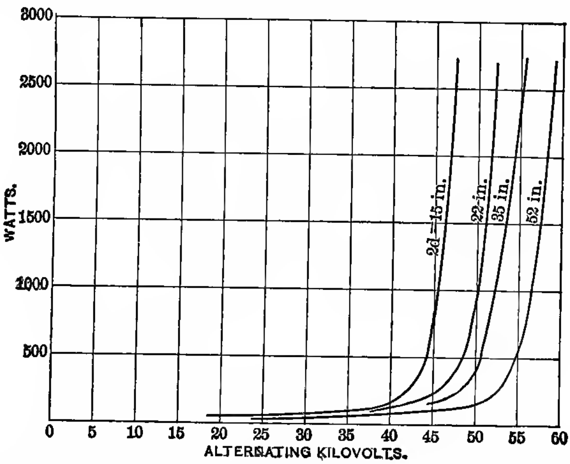


FIG. 134.—Curves showing watts lost in corona on a 2½-mile experimental high-pressure power-transmission line at various pressures and distances between wires of .162 inch diameter. The points marked on the curves near the base were located by equation 154.*

Thus the character of the variation of pressure applied between the conductors with respect to the power lost as found by Mershon and given in Fig. 134 is accounted for. When

* See foot-note on p. 236.

the dielectric flux density, D_0, at which sufficient rupturing has occurred in the atmospheric zone next to the conductor surface to cause appreciable power-loss is known, the following relation must exist between the applied pressure, the diameter of the conductors and their distances apart:

The capacity of one wire, i.e., its capacity with reference to the neutral plane (see equations 101, 142, 145, and 146), is

$$C = 2\pi k \frac{1}{\log_\epsilon \left(\frac{2d}{r}\right)}.$$

The circumference of the wire =

$$2\pi r,$$

and the dielectric flux density immediately outside the surface of the round conductor or wire is

$$D_0 = \frac{EC}{2\pi r}.$$

Substituting,

$$D_0 = \frac{kE}{r \log_\epsilon \left(\frac{2d}{r}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (152)$$

where D_0 is the average flux density about the surface of the conductor. As the conductors on transmission lines are separated more than forty diameters, the dielectric flux is practically uniform on all sides next to the surface of the conductor, and the corona will start on all sides at practically the same pressure. The pressure at which the corona or dielectric conduction-loss occurs on a line of conductors of given diameters and distance apart is, therefore,

$$E = \frac{r \log_\epsilon \left(\frac{2d}{r}\right) D_0}{k}, \quad . \quad . \quad . \quad . \quad . \quad (153)$$

where D_0 is the lowest dielectric flux density in the atmosphere at given barometric pressure at which appreciable power is lost.

There is no need for analyzing this phenomenon further, so

as to obtain the subsequent relation between the magnitude of the power lost and applied pressure, radius of conductors and distance apart, for the reason that such power increases so rapidly with the applied pressure that it will not pay under any circumstances to elevate the pressure to or beyond the point where this power-loss begins.

The law given by equation (153) determines the maximum pressure that may be applied between a conductor having a given radius and its neutral plane, in order that not more than an appreciable loss of power from dielectric conduction may occur. In practice a lower value should be used.

For convenience, equation (153) is rewritten thus:

$$E_{\text{max.}} = 2.055 r \log_{10}\left(\frac{s}{r}\right) D_0 \times 10^{13},* \quad . \quad . \quad (154)$$

where

$E_{\text{max.}}$ = maximum pressure applied between the conductors;
r = radius of the conductors in inches;
s = separation between centres of conductors in inches;
D_0 = lowest dielectric flux at which appreciable power is wasted, in the coulomb-inch-volt system of units.

Mershon's results as given by the curves in Fig. 134 may be used to test this law as follows:

The 52″ curve shows that appreciable power-waste was evident at a pressure of 52,000 volts, alternating; the diameter of the conductor was .162 inch, and their distance apart 15 inches. Substituting these values in equation 154, the value of D_0 is found to be

$$D_0 = \frac{1.71 \times 39{,}200}{2.055 \times .081 \times \log_{10}\left(\frac{15}{.081}\right) \times 10^{13}} \dagger$$

$$= 190{,}500 \times 10^{-13}.$$

* $2.055 \times 10^{13} = \frac{2}{\log^{\epsilon}_{10} \times k}$.

† 1.71 is the form factor of the pressure wave in lieu of $\sqrt{2}$; see Scott, A. I. E. E. Trans., Vol. XV, p. 550.

When this value of D_0 is applied in equation (154) and the various distances between the conductors given in Fig. 134, 15, 22, and 35 inches, together with the values for the radius and diameters of the conductors, .081 and .162 inch, are there substituted, the following values of E at which the power waste begins are obtained:

d, inches.	E. A. C. e.m.f. at which corona loss begins as determined by equation (154).
15	42,100
22	45,100
35	48,900
52	52,000

The points on the curves of Fig. 134 corresponding to these e.m.f.s occupy positions in each instance in that portion where the sudden rise in the power loss occurs. The results of Mershon's measurements, therefore, prove the law given by equation (154) in regard to the relation of the electric pressure to the distance between the conductors at which initial corona loss occurs.*

In conclusion it should be noted that, in dealing with very high pressures for the economical transmission of power over long distances, care should be exercised at all points to see that the outer diameters of the conductors shall be large enough so as to avoid all corona-loss.†

* Mershon's measurements that determine the curves in Fig. 134 were made in 1897–1898 at high altitude where the barometric pressure is about 20 inches of mercury. In a recent, 1903, investigation made by one of the authors it was found that the value of D_0, at which corona or atmospheric conduction loss begins, varies directly as the density of the atmosphere; it varies, therefore, as barometric pressure and temperature vary the atmospheric density. It was also found that D_0 varies irregularly with variation in the diameter of the conductor for diameters less than one-quarter of an inch, while it is constant for larger diameters. The results of this investigation are given in section 4 of the appendix.

† See section 4 of the appendix.

APPENDIX.

1. COPPER WIRE TABLE.*

Am. Gauge, B. & S. No.	Diam., Mils.	Area, Circular Mils (d^2). 1 Mil = .001 inch.	Copper 75° Fahr.			
			R. Ohms per 1000 Feet.	Feet per Ohm.	Ohms per Pound.	Feet per Pound.
0000	460.000	211600.00	.04906	20383.	.000076736	1.561
000	409.640	167805.00	.06186	16165.	.00012180	1.969
00	364.800	133079.40	.07801	12820.	.00019423	2.482
0	324.860	105534.00	.09831	10172.	.00030772	3.130
1	289.300	83694.20	.12404	8062.3	.00048994	3.947
2	257.630	66373.00	.15640	6393.7	.00078045	4.977
3	229.420	52634.00	.19723	5070.2	.0012406	6.276
4	204.310	41742.00	.24869	4021.0	.0019721	7.914
5	181.940	33102.00	.31361	3188.0	.0031361	9.980
6	162.020	26250.50	.39546	2528.7	.0049868	12.58
7	144.280	20816.00	.49871	2005.2	.0079294	15.87
8	128.490	16509.00	.62881	1590.3	.012608	20.01
9	114.430	13094.00	.79281	1261.3	.020042	25.23
10	101.890	10381.00	1.	1000.0	.03182	31.82
11	90.742	8234.00	1.2607	793.18	.050682	40.12
12	80.808	6529.90	1.5898	629.02	.080585	50.59
13	71.961	5178.40	2.0047	498.83	.12841	63.79
14	64.084	4106.80	2.5260	395.97	.2034	80.44
15	57.068	3256.70	3.188	313.85	.3234	101.4
16	50.820	2582.90	4.0191	248.81	.51501	127.9
17	45.257	2048.20	5.0683	197.30	.81900	161.3
18	40.303	1624.30	6.3911	156.47	1.3023	203.4
19	35.890	1288.10	8.0552	124.14	2.067	256.5
20	31.961	1021.50	10.163	98.401	3.2926	323.4
21	28.462	810.10	12.815	78.037	5.2355	407.8
22	25.347	642.70	16.152	61.911	8.3208	514.2
23	22.571	509.45	20.377	49.106	13.238	648.4
24	20.100	404.01	25.695	38.918	21.050	817.6
25	17.900	320.40	32.400	30.864	33.466	1031.
26	15.940	254.01	40.868	24.469	53.083	1300.
27	14.195	201.50	51.519	19.410	84.644	1639.
28	12.641	159.79	64.966	15.393	134.56	2067.
29	11.257	126.72	81.921	12.207	213.96	2607.
30	10.025	100.50	103.30	9.6812	340.25	3287.
31	8.928	79.71	129.63	7.6818	539.55	4145.
32	7.950	63.20	164.26	6.0880	860.33	5227.
33	7.080	50.13	207.08	4.8290	1367.3	6591.
34	6.304	39.74	261.23	3.8281	2175.5	8311.
35	5.614	31.52	329.35	3.0363	3458.5	10480.
36	5.000	25.00	415.24	2.4081	5497.4	13210.
37	4.453	19.83	523.76	1.9093	8742.1	16660.
38	3.965	15.72	660.37	1.5143	13899.	21010.
39	3.531	12.47	832.48	1.2012	22047.	26500.
40	3.144	9.89	1049.7	.9527	35055.	33410.

* Copper: Resistance of one mil-foot of pure soft copper at 75° F. is 10.381 ohms; resistance of hard copper is 1.0226 times that of soft copper. Weight in pounds per mil-foot .000003028. (By permission of the Pittsburg Reduction Co.)

ALUMINUM WIRE TABLE.*

Am. Gauge, B. & S. No.	Diam., Mils.	Area, Circular Mils (d^2). 1 Mil = .001 inch.	Aluminum 75° Fahr.			
			R. Ohms per 1000 Feet.	Feet per Ohm.	Ohms per Pound.	Feet per Pound.
0000	460.000	211600.00	.08177	12229.8	.00042714	5.185
000	409.640	167805.00	.10310	9699.0	.00067022	6.539
00	364.800	133079.40	.13001	7692.0	.00108116	8.246
0	324.860	105534.00	.16385	6245.4	.0016739	10.397
1	289.300	83694.20	.20672	4637.35	.0027272	13.108
2	257.630	66373.00	.26077	3836.22	.0043441	16.529
3	229.420	52634.00	.32872	3036.12	.0069057	20.846
4	204.310	41742.00	.41448	2412.60	.0109773	26.281
5	181.940	33102.00	.52268	1913.22	.017456	33.146
6	162.020	26250.50	.65910	1517.22	.027758	41.789
7	144.280	20816.00	.83118	1203.12	.044138	52.687
8	128.490	16509.00	1.06802	964.18	.070179	66.445
9	114.430	13094.00	1.32135	756.78	.111561	83.822
10	101.890	10381.00	1.66667	600.00	.17467	105.68
11	90.742	8234.00	2.1012	475.908	.28211	133.24
12	80.808	6529.90	2.6497	377.412	.44856	168.01
13	71.961	5178.40	3.3412	299.298	.71478	211.86
14	64.084	4106.80	4.3180	231.582	1.16225	267.17
15	57.068	3256.70	5.1917	192.612	1.7600	336.93
16	50.820	2582.90	6.6985	149.286	2.8667	424.81
17	45.257	2048.20	8.4472	118.380	4.5588	535.62
18	40.303	1624.30	10.6518	93.882	7.2490	675.67
19	35.890	1288.10	13.8148	72.384	12.1916	851.79
20	31.961	1021.50	16 938	59.0406	18.328	1074.11
21	28.462	810.10	21.358	46.8222	29.142	1354.65
22	25.347	642.70	26.920	37.1466	46.316	1707.94
23	22.571	509.45	33.962	29.4522	73.686	2153.78
24	20.100	404.01	42.825	23.3508	117.170	2715.91
25	17.900	320.40	54.000	18.5184	186.28	3424.66
26	15.940	254.01	68.113	14.6814	296.32	4317.78
27	14.195	201.50	85.865	11.6460	485.56	5446.63
28	12.641	159.79	108.277	9.2358	749.02	6868.13
29	11.257	126.72	136.535	7.3242	1190.97	8698.03
30	10.025	100.50	172.17	5.8087	1893.9	10917.0
31	8.928	79.71	212.12	4.7144	2941.5	13762.8
32	7.950	63.20	273.97	3.6528	4788.9	17361.1
33	7.080	50.13	345.13	2.8974	7610.7	21886.7
34	6.304	39.74	435.38	2.2969	12109.4	27609.1
35	5.614	31.52	548.92	1.8218	19251.	34807.3
36	5.000	25.00	692.07	1.4449	30600.	43878.9
37	4.453	19.83	872.93	1.1456	48661.	55340.4
38	3.965	15.72	1100.62	.9086	76658.	69783.7
39	3.531	12.47	1387.47	.7207	121881.	88028.2
40	3.144	9.89	1749.50	.5716	193835.	111099.0

* Aluminum: Resistance of one mil-foot of pure aluminum at 75° F. is 17.303 ohms. Weight in pounds per mil-foot .0000009115. (By permission of the Pittsburg Reduction Co.)

2. The student should keep clearly in mind the relation of the *ampere-inductor* and the *ampere-turn.* The m.m.f. that an ampere-inductor will exert through a magnetic circuit that surrounds it is precisely the same as that of an ampere-turn. The inductor in any case is but a part of a complete electric-current circuit which surrounds the magnetic circuit once for each inductor that is enveloped by such magnetic circuit.

The student can check the above statement of this fact for himself by noting in connection with any of the cylinder field windings and their magnetic circuits that when the circuit includes but one inductor per individual magnetic circuit, the corresponding electric circuit has made but one turn around that particular magnetic circuit, no matter how many other inductors the circuit may have been taken through in so doing; likewise he may note that where a current circuit passes through two or more inductors that are surrounded by a single magnetic circuit, it makes two or more complete turns around such magnetic circuit regardless of the number of other inductors surrounded by other magnetic circuits through which the current circuit may have been completed

3. $$E_{\text{ave.}} = 4 \times f \times B_{\text{max.}} \pi r_i^2 \times 10^{-8}.$$

The factor 4 indicates that the total flux cuts the circuit 4 times per cycle.

$$E_{\text{eff.}} = E_{\text{ave.}} \times \frac{\pi}{2} \times \frac{1}{\sqrt{2}},$$

$$E_{\text{eff.}} = 4 \times f \times B_{\text{max.}} \pi r_i^2 \times \frac{\pi}{2} \times \frac{1}{\sqrt{2}} \times 10^{-8},$$

$$= 2\pi f \left(\frac{1}{\sqrt{2}} \pi r_i^2 B_{\text{max.}} \times 10^{-8} \right).$$

4. In an investigation conducted by one of the authors the following facts were found regarding the variation of the dielectric flux density of the atmosphere surrounding the conductors

of a high alternating pressure line at which corona or atmospheric conduction occurs:

The atmosphere about the conductor has the least dielectric strength at a radial distance from the center of the conductor that is a small amount greater than the radius of the conductor. The molecular forces are such as to cause the zone of atmosphere in direct contact with the conductor to possess greater dielectric strength than the zones beyond. Thus, while the zone in immediate contact with the conductor is subjected to a greater dielectric flux density, it is not the first to rupture owing to its superior dielectric strength.

For a conductor having a diameter of *one-quarter inch or greater*, the atmospheric zone that first ruptures and becomes conductive is located at a constant distance of .07 inch from the surface of the conductor. For such conductor, therefore, the radius of the atmospheric zone that first ruptures is the *radius of the conductor plus* .07 *inch.*

For this class of conductors the dielectric flux density at which the weakest atmospheric zone ruptures remains constant.

For diameters *smaller than one-quarter inch* the distance from the conductor surface at which the weakest atmospheric zone occurs grows less as the conductor diameter diminishes; the dielectric flux density at which the weakest zone ruptures increases as the conductor diameter decreases.

The table below gives the corresponding values of radii of the weakest atmospheric zones and the dielectric flux densities required to rupture such zones, wherein the following notation is used:

r, radius of conductor in inches;

r', radius of weakest atmospheric zone in inches;

$r'-r$, thickness of weakest atmospheric zone in inches;

D_0', dielectric flux density in coulombs per square inch at which the weakest atmospheric zone ruptures;

D_0, dielectric flux density at surface of conductor corresponding to D_0'.

Diameter in Inches.	r	r'	$r'-r$	D_0'	D_0
.05	.0250	.0348	.0098	$305,000 \times 10^{-13}$	$425,000 \times 10^{-13}$
.15	.075	.1016	.0266	$240,000 \times 10^{-13}$	$325,000 \times 10^{-13}$
.20	.1	.166	.066	$172,000 \times 10^{-13}$	$285,000 \times 10^{-13}$
.25	.125	.195	.07	$170,000 \times 10^{-13}$	$265,000 \times 10^{-13}$
.35	.175	.245	.07	$170,000 \times 10^{-13}$	$238,000 \times 10^{-13}$
.45	.225	.295	.07	$170,000 \times 10^{-13}$	$223,000 \times 10^{-13}$
.55	.275	.345	.07	$170,000 \times 10^{-13}$	$213,000 \times 10^{-13}$
.65	.325	.395	.07	$170,000 \times 10^{-13}$	$206,000 \times 10^{-13}$

Barometer 29.5 inches. Temperature 70° Fahrenheit.

It was found that the values of D_0 and D_0' as given in the above table varied in direct proportion to the density of the atmosphere throughout the ranges of barometric pressure and temperature in which the measurements were made, viz.:

18 to 36 inches of mercury, and
70° to 200° Fahrenheit.

On page 234 it was shown that

$$D_0 = \frac{EC}{2\pi r} = \frac{kE}{r \log_\varepsilon \left(\frac{2d}{r} \right)},$$

where $2\pi r$ is the circumference of the conductor which was there assumed to be the circumference of the zone of atmosphere that is first broken down, becoming conductive. From the above facts it is seen that this circumference is $2\pi r'$, in lieu of $2\pi r$, wherein r' is a fraction of an inch, dependent upon the size of the conductor, larger than r. Equation (154) becomes, therefore,

$$E_{\text{max.}} = 2.055 r' \log_{10} \left(\frac{s}{r} \right) D_0' \times 10^{13}. \quad . \quad . \quad (154a)$$

For conductor diameters of *one-quarter inch and upwards*

$r' = r + .07$, and

$D_0' = 170{,}000 \times 10^{-13}$ at 70° F. and 29.5 inches barometer, and for which equation (154*a*) becomes

$$E_{\text{max.}} = 350{,}000(r + .07)\log_{10}\left(\frac{s}{r}\right) \quad . \quad . \quad (154b)$$

at 70° F. and 29.5 inches barometer.

Since the value of D_0 varies directly as the density of the atmosphere, a corresponding factor must be introduced in equations (154) and (154*a*) where values of $E_{\text{max.}}$ are to be determined for barometric pressures and temperatures other than

29.5″ B. and 70° F.,

at which the values of D_0 given in the above table were determined.

The weight of a cubic foot of atmosphere at any temperature and barometric pressure is

$$W = \frac{1.3253 \times b}{459.2 + t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (*)$$

wherein b = height of barometer in inches;

t = temperature in degrees Fahrenheit; $459.2 + t$, corresponding absolute temperature;

1.3253 = weight in lbs. of 459.2 cu. ft. o air at 0° F. and at one inch of barometric pressure.

Reduction factor for atmospheric density due to any temperature and barometric pressure:

$$\frac{1.3253b}{459.2 + t} \div \frac{1.3253 \times 29.5}{459.2 + 70} = \frac{17.94b}{459.2 + t}.$$

* Kent's Mechanical Engineer's Pocket-book: Properties of Air.

It follows, therefore, that when equations (154*a*) and (154*b*) are written so as to include the factor that accounts for changes in temperature and barometric pressure they become

$$E_{\text{max.}}=\frac{17.94b}{459.2+t}\times 2.055r' \log_{10}\left(\frac{s}{r}\right) D_0'\times 10^{13}. \quad (154a')$$

$$E_{\text{max.}}=\frac{17.94b}{459.2+t}\times 350,000(r+.07) \log_{10}\left(\frac{s}{r}\right). \quad . \quad (154b')$$

In order to realize something of the practical meaning of these laws proper substitutions were made in equations (154) and (154*b*) to determine the diameters of conductors required to operate without atmospheric conduction loss on a high-pressure line at various pressures ranging from 50,000 to 250,000 volts at a separation of 48 inches, a temperature of 70° F., and a barometric pressure of 29.5 inches of mercury.

For diameters less than .25 inch equation (154) was used and the corresponding values of r and D_0 were taken from the above table and substituted therein to determine values of $E_{\text{max.}}$ at which atmospheric loss would occur. The values of $E_{\text{max.}}$ for diameters larger than .25 inch were determined by substitution in equation (154*b*), the corresponding values of r.

In this manner a series of values of $E_{\text{max.}}$ was determined corresponding to the series of assumed values of conductor radii.

A curve was then located using the values of r and $E_{\text{max.}}$ as coordinates, showing the relation between the size of the conductors and maximums of pressure-waves at which loss of power by atmospheric conduction would occur. From this curve values of r and their corresponding values of $E_{\text{max.}}$ were taken and placed in the table given below, where there are given also the effective pressures having 90% of the values of the foregoing pressures at which the atmospheric conductive loss would occur. The latter pressures are about the highest

that could be used with their corresponding sizes of conductors without encountering the corona loss. The pressures were selected so as to cover a range of 50,000 to 250,000 effective sine-wave volts.

Maximum Volts at which Atmospheric Conduction Loss Occurs.	Corresponding Effective Volts, (Sine Wave).	Operating Pressure 90 Per Cent. of Corresponding Effective Volts.	Diameter of Conductors in Inches.
78,500	55,500	50,000	.058
118,000	83,300	75,000	.106
157,000	111,100	100,000	.192
235,500	166,600	150,000	.430
314,000	222,200	200,000	.710
392,000	277,700	250,000	.990

Distance between centers of conductors, 48 inches; temperature, 70° Fahr.; barometric pressure, 29.5 inches of mercury.

Dr. Blaker studied the conductivity of the atmosphere with continuous high pressures when the molecular structural character of the atmosphere is unimpaired, *i.e.*, when all corona or "ionization," *i.e.*, breaking up of the atomic structure of the atmosphere, is absent and when the atmosphere is free of conducting material, as, for example, smoke, that it does not conduct a continuous current, *i.e.*, the atmosphere behaves as a true dielectric so long as the dielectric flux to which it is subjected does not form corona.

In the future, therefore, when practical necessities demand, very high pressures will be used for long-distance transmission of power. It is seen in the last table above that with a separation of 48 inches between conductor centres the diameters of the conductors will have to be about *one inch* in order to transmit without atmospheric loss at a pressure of 250,000 effective sine-wave volts at ordinary temperatures and barometric pressures. Over mountains larger sizes must be used on account of the lower barometric pressures encountered. The highest temperatures to be encountered are not likely to exceed 100° Fahr. or a rise of 30° above the normal taken at 70°, which on the absolute

scale amounts to a rise of about 7% and which means that the maximum of the pressure wave at which corona will first form for a given set of conditions will be lowered by 7% when the temperature is elevated from 70° to 100° F.

Power can probably be transmitted at these very high pressures, 250,000 volts, only on a scale sufficiently large to warrant the expense of building a cover over the line so as to keep snow, rain, and dust away from it and then only after the electrical manufacturing knowledge and art can produce practical transformers for the production of electric power in large amounts at such high pressures.

Where the diameter of the solid or stranded conductor required to avoid corona is larger than that required to conduct the current economically the conductor may be made tubular.

The electric pressures considered above have been maximum instantaneous pressures or effective values of sine-wave pressures.

Equations (154), (154a), (154b), (154a'), and (154b') were written so as to give maximum instantaneous values. These values divided by $\sqrt{2}$ will render sine-wave effective values. For all cases, however, where the wave form of the line pressure differs from the sine wave the *maximum-to-effective factor* for the particular wave under consideration must be known and must be used in lieu of the $\sqrt{2}$, which is practically applicable only when the form of the wave is known to be a very close approximation to the sine wave.

The constants of a high-pressure circuit relating to the inductance and capacity of the line and transformers and the armature reaction of the generator are such as to modify the sine pressure wave in most cases materially, thus causing in general the maximum-to-effective factor to be different from $\sqrt{2}$, requiring correspondingly conductors having larger or smaller diameters, according as this factor is greater or less than the $\sqrt{2}$.

From equation (154a') it is obvious that in all cases where

electrical circuits are to be insulated for high pressures and where the atmosphere or other gases cannot be entirely displaced from the immediate region of such circuits by solid or liquid dielectrics, as is generally the case, the practical value of the total or composite insulation must be greatly increased by enclosing such electric circuit in tight and strong compartments capable of sustaining a high pneumatic or hydrostatic pressure according as dry or liquid insulation is employed. Such high pressure should be applied for the purpose of greatly increasing the strength of the atmosphere or other gas that may be confined in the presence of the solid or liquid dielectric which constitutes the main insulation.

The compression of the air or gas in the presence of the insulation will greatly improve the practical dielectric strength of the insulation of such high-pressure circuits until it is carried to a point where the dielectric strength of the air and gas to resist rupture and corona formation is equal to or greater than the dielectric strength of the main solid or liquid insulation. At ordinary temperatures and barometric pressures the alternating dielectric flux that an insulation will stand is limited to that at which the included air or gas is broken; the heating thereby caused, rapidly effects the destruction of the main insulation. (See Sec. 39.) Mechanical pressure will increase the dielectric strength of the air or gas and correspondingly increase the practical working strength of the main insulation.

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